Information Dynamics and Debt Maturity*

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Abstract

I develop a dynamic model of financing decisions and optimal debt maturity choice in which creditors face adverse selection and learn about the firm's quality from news. In equilibrium, shareholders may choose to postpone debt issuance to reduce adverse selection and improve the pricing of newly issued debt. Over time, the benefits of learning decrease and zero-leverage firms eventually decide to issue debt. Because shorter maturity debt is less sensitive to information, younger firms issue shorter maturity debt to alleviate adverse selection while mature firms issue longer maturity debt, leading to a life-cycle theory of debt maturity.

Keywords: debt maturity; capital structure; adverse selection; zero leverage; debt issuance.
JEL classification: G32; D82; D83.

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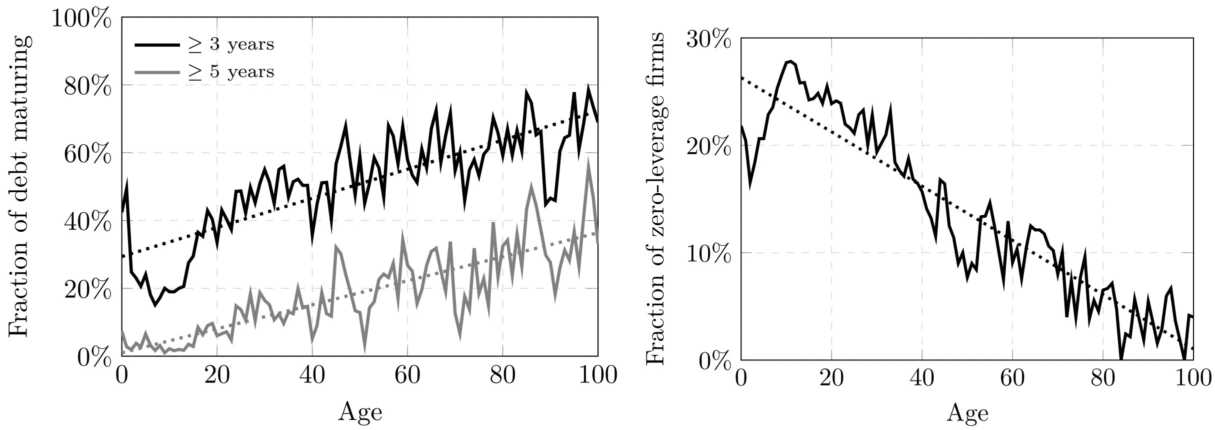
Myers and Majluf (1984)'s seminal work shows that asymmetric information between creditors and a firm influences the firm's financing decisions. This insight has been applied to firms' debt maturity choice and has spawned a large body of theoretical research (see e.g. Flannery, 1986 and Diamond, 1991, 1993). Although we have learned much from this work, virtually all models are static even though firms' debt maturity choice is inherently a dynamic decision.[1] In this article, I contribute to the literature by developing a dynamic model of financing decisions and optimal debt maturity choice in which creditors face adverse selection and learn about the firm's quality over time, which leads to a rich set of new insights and empirical implications.

The central result of this article is that firms lengthen their debt maturity as they mature, leading to a life-cycle theory of debt maturity. Younger firms that face more adverse selection issue shorter maturity debt, while mature firms that face less adverse selection issue longer maturity debt. This life-cycle theory of debt maturity is consistent with Fig. 1(a), which shows that firm age and debt maturity are positively correlated in the data. Furthermore, it is also rationalizes the findings in Barclay and Smith (1995), Berger et al. (2005), and Custódio et al. (2013) that firms facing more asymmetric information issue shorter maturity debt.

In my single-firm model, shareholders issue debt to shield the firm's operating income from taxes. Competitive creditors cannot observe firm quality, which is either high or low, and determines the firm's survival probability. Creditors thus face adverse selection when buying the firm's debt. Therefore, creditors undervalue a high-quality firm's debt and overvalue a low-quality firm's debt. Over time, however, creditors learn about firm quality from its survival and debt issuance behavior.

Focussing first on the timing of debt issuance, the firm's incentive to issue debt differ depending on its quality. A high-quality firm can either issue underpriced debt immediately and capture the tax benefits or wait until creditors learn about its quality and issue debt with lower underpricing. A low-quality firm faces a similar trade-off: it can either issue (fairly priced) debt immediately, which reveals its quality, or wait in the hope of mimicking

[1] Firms' debt maturity matters for their real decisions and therefore for their firm value. For example, Almeida et al. (2011) show that firms' debt maturity influences its investment decisions, while Gopalan et al. (2013) show that it affects firms' default decisions.



(a) Firm age versus debt maturity

(b) Firm age versus fraction of zero-leverage firms

FIGURE 1: **The effect of firm age on firms' debt maturity and on the fraction of zero-leverage firms.** Fig. 1(a) shows the median fraction of debt maturing in more than 3 and 5 years conditional on firm age and the firm having debt outstanding. Fig. 1(b) shows the fraction of zero-leverage firms conditional on firm age.[2]

a high-quality firm and sell overpriced debt in the future.

These trade-offs lead to a natural equilibrium in threshold strategies. In this equilibrium, both types of firm issue debt when creditors' beliefs that the firm is high quality are above an upper threshold. A low-quality firm also issues debt when creditors' beliefs that the firm is high quality are below a lower threshold. For beliefs above this upper threshold, a high-quality firm issues debt because the tax benefits outweigh the gain of waiting for creditors to learn, while a low-quality firm mimics a high-quality firm to issue overpriced debt. The further beliefs fall below the upper threshold, the lower are the chances of a low-quality firm reaching this threshold and being able to sell overpriced debt. There exists a point at which a low-quality firm is indifferent between postponing issuance in the hope of selling overpriced debt in the future and issuing debt immediately and thus revealing its quality. This indifference pins down the lower threshold. Below this threshold, a low-quality firm randomizes between issuance and postponing issuance so that it remains indifferent between the two. This happens when beliefs are reflected at the lower threshold upon the firm postponing debt issuance.

[2] I define the fraction of debt maturing in more than 3 and 5 years as (DLTT-DD2-DD3)/(DLTT+DLC) and (DLTT-DD2-DD3-DD4-DD5)/(DLTT+DLC). A firm is a zero-leverage firm if DLTT+DLC=0. I calculate firm age using founding year from Jay Ritter's IPO date data. The figures use the Compustat sample from 1987 until 2014.

Consider next the debt maturity. The firm faces fixed issuance costs each time it issues debt, and therefore in the absence of asymmetric information the firm wants to issue long-term (perpetual) debt to minimize these costs. In the presence of asymmetric information, a high-quality firm has two reasons to shorten the debt maturity. First, shorter maturity debt faces less adverse selection because the likelihood of default increases with debt maturity. Second, when debt matures creditors have updated their beliefs about firm quality, which lowers the mispricing of future debt issues. When choosing the debt maturity, shareholders balance the benefits of a shorter maturity against the fixed cost of issuing debt.

Above the upper threshold, the optimal debt maturity is increasing in beliefs. The higher creditors' beliefs are, the lower the uncertainty about firm quality and therefore the smaller the adverse selection problem. Thus, issuance costs become relatively more important and hence shareholders lengthen the debt's maturity to decrease the frequency with which they incur these costs. At or below the lower threshold, issuance reveals firm quality and asymmetric information no longer plays any role. Thus, to minimize issuance costs, a low-quality firm issues long-term (perpetual) debt. This equilibrium leads to a life-cycle theory of debt maturity: Younger firms that face more adverse selection issue shorter maturity debt, while mature firms that face less adverse selection, when creditors are more certain the firm is either high or low quality, issue longer maturity debt.

Another important result of this article is the existence of zero-leverage firms in equilibrium. Given the large net benefits to debt, which according to Korteweg (2010) are around 5.5% of firm value for the median firm, the existence of zero-leverage firms has puzzled economists. There is still no satisfying explanation for why around 20% of US firms remain unlevered.[3] In my model, for beliefs in between the two issuance thresholds, the firm postpones debt issuance and has zero leverage. Furthermore, these zero-leverage firms are expected to issue debt in the future, when beliefs reach one of the issuance thresholds, and therefore are worth more than permanently unlevered firms. This result is consistent with Korteweg (2010)'s finding that zero-leverage firms have expected net benefits to debt. Finally, because of learning, the model also predicts that uncertainty about a firm's quality

[3]See Strebulaev and Yang (2013) for evidence on the existence of the zero-leverage phenomenon in the US and Bessler et al. (2013) for the existence of the zero-leverage phenomenon in the rest of the world.

decreases with its age and so does the probability that a firm has zero leverage. This result is consistent with Fig. 1(b), which shows that the zero-leverage phenomenon is more prevalent among younger firms.

I also study the effect of firms' debt maturity choice on the efficiency of the equilibrium, which I define as firm value relative to the first-best firm value. The first-best firm value is the firm value in case the firm directly issues long-term debt. Giving the firm the opportunity to dynamically manage its debt maturity, instead of being able to issue debt with only one maturity, can both benefit and harm efficiency. The reason is that both high and low-quality firms set their debt maturity to maximize their equity value. When creditors believe that the firm is more likely to be of low quality, giving the firm the ability to manage its debt maturity increases efficiency. The reason is that the firm's ability to shorten its debt maturity, and thereby alleviating adverse selection, speeds up debt issuance, which increases efficiency. When creditors believe the firm is more likely to be of high quality, this is not necessarily the case. In this situation, the firm issues debt independent of whether or not it can select its debt maturity. In case the firm can select its debt maturity, the firm selects shorter maturity debt to alleviate the adverse selection problem. Restricting the firm to issue only long-term debt would improve efficiency because it minimizes issuances costs. This shows that giving firms the ability to dynamically manage their debt maturity can be a blessing and a curse.

To extend the analysis, I add to the model an exogenous Brownian news process with an informative drift and show numerically that the results previously found are robust. The trade offs described above still exist in this setup, and therefore the equilibrium described above still exists. This shows that the life-cycle theory of debt maturity is robust to incorporating this news process, as are the results on zero-leverage firms. Furthermore, in the model with a Brownian news process there is a positive probability that a levered firm that issued finite maturity debt becomes a zero-leverage firm, which is consistent with the data as is shown later in this article.

The article is organized as follows. Section I discusses the related literature. Section II describes the model. Section III constructs an equilibrium and Section IV studies the model's implications. Section V incorporates an exogenous Brownian news process into the model and studies its implications. Section VI concludes. All proofs are in the appendix.

I. Related Literature

This article incorporates asymmetric information into a dynamic capital structure model to study firms' debt maturity and timing of debt issuance. I combine a dynamic capital structure model with Daley and Green (2012)'s asset market model with dynamic adverse selection. I also endogenize shareholders' debt maturity choice, something which is not present in Daley and Green (2012)'s framework. Furthermore, while Daley and Green (2012, 2016) look at the asset pricing implications of dynamic adverse selection, I study how it influences firms' debt maturity choice and timing of debt issuance.

Myers and Majluf (1984) seminal work uses asymmetric information to explain financing decisions. Strebulaev et al. (2016), Hennessy et al. (2010), and Morellec and Schürhoff (2011) develop dynamic versions of Myers and Majluf (1984). None of these three articles looks at firms' (dynamic) debt maturity choice. Furthermore, they focus on financially constrained firms that need to raise funding to finance investment, while in my model firms issue debt for tax reasons. This allows me, for example, to study zero-leverage firms that make payouts to their shareholders, which are financially unconstrained. Strebulaev et al. (2016) also use Daley and Green (2012)'s framework. In their model, the firm's realized cash flow has two effects. First, it increases the firm's internal funds and second investors learn about the firm's quality. Both effects lower adverse selection when financing investment. In this setting, they study the implications of dynamic adverse selection for security design and announcement returns. Hennessy et al. (2010) and Morellec and Schürhoff (2011) develop dynamic signaling models in which shareholders need to decide on a firm's financing and investment.[4] In Hennessy et al. (2010) and Morellec and Schürhoff (2011), investors learn only from the actions taken by the firm and not from the firm's survival or news, as is the case in my model.

This article also endogenizes firms' debt maturity choice. Flannery (1986), Diamond (1991, 1993), and Goswami et al. (1995) develop static models that study the implications of asymmetric information on firms' debt maturity choice. Because my model is dynamic, I can

[4] Leland and Pyle (1976) and Ross (1977) where among the first to introduce signaling models in corporate finance.

develop a life-cycle theory of debt maturity. Furthermore, my model combines this dynamic debt maturity choice with a decision on the timing of debt issuance instead of examining these two choices separately.

To the best of my knowledge, this article develops the first dynamic debt maturity model in which asymmetric information drives the choice of debt maturity. Brunnermeier and Oehmke (2013), He and Milbradt (2016), Geelen (2016), Ju and Ou-Yang (2006), Crouzet (2016), and Huang et al. (2019), among others, develop dynamic debt maturity models in perfect information settings. Geelen (2016) incorporates debt with an exponentially distributed maturity date into a Leland (1994) model. He studies the effects of the flexibility to alter the firm's leverage and debt maturity when the debt matures on firms' optimal leverage and debt maturity. As in Geelen (2016), I model the maturity date as an exponentially distributed time.

Existing dynamic capital structure models (Leland, 1994, 1998) have a hard time explaining the zero-leverage phenomenon.[5] The reason is that in the absence of (sufficiently large) fixed issuance cost shareholders always have the incentive to issue debt to shield the firm's operating income from taxes. In my model, shareholders opt for zero leverage because of adverse selection and learning about the firm's quality. Most existing dynamic capital structure models assume perfect information and are therefore unable to generate zero-leverage firms.[6]

[5] Agrawal and Nagarajan (1990), Strebulaev and Yang (2013), Bessler et al. (2013), Devos et al. (2012), and El Ghoul et al. (2018) empirically study the zero-leverage phenomenon. They are unable to find consensus on what drives firms' zero leverage choice. Agrawal and Nagarajan (1990) and Strebulaev and Yang (2013) claim that managerial preferences and family firms are driving the zero-leverage phenomenon. Strebulaev and Yang (2013) also find that the nature of the investment and cash flow process is an important determinant. Devos et al. (2012) and Bessler et al. (2013) attribute the zero-leverage phenomenon to financial constraints and El Ghoul et al. (2018) to culture. Most of these studies find results consistent with asymmetric information influencing firms' zero leverage choice.

[6] Other theories try to explain the zero-leverage phenomenon. These theories are, for example, related to human capital (Berk et al. (2010) and Lambrecht and Pawlina (2013)), information production (Chang and Yu (2010)), and investment opportunities (Hackbarth and Mauer, 2012 and Sundaresan et al., 2015). All these mechanisms differ from mine. Furthermore, most of these theories have a hard time explaining Korteweg (2010)'s finding that zero-leverage firms are expected to issue debt in the future

II. Model

A. Perfect Information

The probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ represents the uncertainty in the economy. Shareholders and creditors are risk-neutral and discount cash flows at the risk-free rate $r > 0$. The firm acts in the best interest of its shareholders.

The firm generates a before tax operating income X_t. This operating income is one until an exponentially distributed time $\tau_F \sim \exp(\lambda_\theta)$ at which point it becomes zero and the firm fails so that

$$X_t = \mathbb{1}_{\{t<\tau_F\}}.$$

The (failure) intensity of the exponentially distributed time is λ_θ. When I introduce asymmetric information, θ indicates the firm's quality. This definition of the operating income implies that firm value is zero after the firm fails at τ_F. The operating income X_t is a special case of Abel (2017)'s operating income with the new operating income drawn from the probability distribution $\mathbb{P}^\theta(X_{\tau_F} = 0) = 1$. Exponentially distributed times are regularly used to model firms or projects' cash flows, see for example Biais et al. (2010), He (2012), Myerson (2015), Green and Taylor (2016), Strebulaev et al. (2016), and Varas (2018).[7] The tax rate is π, and the firm's after tax operating income is $1-\pi$ until it fails. Therefore, under perfect information the unlevered firm value is

$$\textit{Unlevered Firm Value} = \mathbb{E}_0^\theta\left[\int_0^{\tau_F} e^{-rt}(1-\pi)dt\right] = \frac{(1-\pi)}{r+\lambda_\theta}.$$

The expectations operator $\mathbb{E}_t^\theta$ is the expectation given the information up to time t conditional on θ, $\mathbb{E}_t^\theta[X] = \mathbb{E}[X|\mathcal{F}_t, \theta]$. Similarly, the expectations operator $\mathbb{E}_t$ is the expectation given the information up to time t not knowing θ, $\mathbb{E}_t[X] = \mathbb{E}[X|\mathcal{F}_t]$.

The firm can issue debt with coupon c to reduce corporate taxes. As in Geelen (2016), I model the debt maturity date as an exponentially distributed time $\tau_m \sim \exp(1/m)$ and it

[7] I use this Poisson cash flow structure to keep the model tractable. This cash flow structure also implies that I do not incorporate investment opportunities into the model. Although this assumption is restrictive, the driving forces described in this framework would still exist in a richer framework.

thus has expectation

$$\mathbb{E}_0^\theta\left[\tau_m\right] = m.$$

For simplicity, the debt contract has no principal.[8] The value of debt with coupon $c \leq 1$ is

$$Debt\ \ Value = \mathbb{E}_0^\theta\left[\int_0^{\tau_F \wedge \tau_m} e^{-rt} c dt\right] = \frac{c}{r + \lambda_\theta + \frac{1}{m}}.$$

Creditors receive coupon payments c until either the firm fails at τ_F, or until the debt matures at τ_m.

The equity value is

$$Equity\ \ Value = \mathbb{E}_0^\theta\left[\int_0^{\tau_F \wedge \tau_m} e^{-rt}(1-\pi)(1-c)dt + \mathbb{1}_{\{\tau_m < \tau_F\}} e^{-r\tau_m}(Firm\ \ Value)\right].$$

Shareholders receive the after tax operating income minus coupon payments $(1-\pi)(1-c)$ until either the debt matures at τ_m or the firm fails at τ_F. If the debt matures before the firm fails, shareholders own the firm's future operating income again. If the firm fails before the debt matures, equity value drops to zero.

Debt issuance is costly. Notably, shareholders incur a fixed cost q when issuing debt. Under perfect information, shareholders want to minimize issuance costs and thus issue perpetual debt. Furthermore, shareholders optimally pick a coupon $c = 1$ to shield the firm's operating income from taxes. The optimal firm value is

$$\begin{aligned}
Firm\ \ Value &= \sup_{m, c \leq 1}\left\{Equity\ \ Value + Debt\ \ Value - q\mathbb{1}_{\{c>0\}}\right\} \\
&= \sup_{m, c \leq 1} \mathbb{E}_0^\theta\left[\int_0^{\tau_F \wedge \tau_m} e^{-rt}\left((1-\pi) + \pi c\right) dt + \mathbb{1}_{\{\tau_m < \tau_F\}} e^{-r\tau_m}(Firm\ \ Value) - q\mathbb{1}_{\{c>0\}}\right] \\
&= \mathbb{E}_0^\theta\left[\int_0^{\tau_F} e^{-rt} dt - q\right] = \frac{1}{r + \lambda_\theta} - q.
\end{aligned}$$

The firm generates a operating income of one that is shielded from taxes until it fails. In this setting, there is no asymmetric information, and the firm is thus able to sell its debt to

[8] Adding a principal that does not lead to strategic default at maturity is a trivial extension because the equilibrium described in the following section would still exist.

competitive creditors and increase its value by

$$\frac{\pi}{r+\lambda_\theta} - q > 0,$$

which I assume is positive.

Because the tax benefits of a single debt issue decrease with the debt maturity while the issuance cost remain constant, there exists a lower bound on the debt maturity below which debt issuance becomes negative NPV.

Proposition 1. *Under perfect information debt issuance is positive NPV for a firm of quality θ if and only if $m > \underline{m}_\theta$, where*

$$\underbrace{\frac{\pi}{r+\lambda_\theta+\frac{1}{m}}}_{\textit{tax benefits}} - \underbrace{q}_{\textit{issuance cost}} > 0 \Leftrightarrow m > \frac{1}{\frac{\pi}{q} - r - \lambda_\theta} = \underline{m}_\theta.$$

To simplify notation, I define the value of debt with coupon $c = 1$ as

$$D_\theta(m) = \mathbb{E}_0^\theta \left[\int_0^{\tau_F \wedge \tau_m} e^{-rt} 1 dt \right] = \frac{1}{r+\lambda_\theta+\frac{1}{m}}$$

and the value of the same cash flow taxed and paid out to shareholders as

$$K_\theta(m) = \mathbb{E}_0^\theta \left[\int_0^{\tau_F \wedge \tau_m} e^{-rt} (1-\pi)\, dt \right] = \frac{1-\pi}{r+\lambda_\theta+\frac{1}{m}}.$$

The tax benefits of debt lead to the gains from debt issuance, $D_\theta(m) > K_\theta(m)$.

In reality, there is asymmetric information about the quality of the firm and over time creditors learn about the quality of the firm. The next two subsections incorporate asymmetric information and learning into the model.

B. Asymmetric Information

Assume there are two types of firms indexed by $\theta \in \{h, l\}$.[9] One type is high quality h and is expected to survive longer than the other type, which is low quality l. This implies that the failure intensity of a high-quality firm is lower than the failure intensity of a low-quality firm: $\lambda_h < \lambda_l$. Shareholders know the firm's quality but creditors do not. For both types, it is optimal to issue debt but under perfect information creditors' valuations differ:

$$D_h(m) > D_l(m).$$

Therefore, asymmetric information creates an adverse selection problem. Over time, creditors learn about the firm's quality from news, its survival, and its debt issuance behavior.

Next, I describe the debt issuance game between shareholders and creditors. To ensure that the adverse selection problem creditors face in this issuance game is severe enough, I make the following assumption

Assumption 1. *The adverse selection problem is severe enough in that*

$$K_h(m) > \frac{r + \lambda_l}{r + \lambda_l + \frac{1}{m}} \left(D_l(\infty) - q \right), \quad \forall m > \underline{m}_h.$$

This assumption has two implications. First, shareholders of a high-quality firm prefer to postpone debt issuance rather than selling debt at a low-quality firm's valuation,

$$K_h(m) > D_l(m) - q, \quad \forall m > \underline{m}_h.$$

Second, shareholders of a low-quality firm benefit from mimicking a high-quality firm instead of separating using perpetual debt,

$$K_h(m) > \frac{r + \lambda_l}{r + \lambda_l + \frac{1}{m}} \left(D_l(\infty) - q \right) = \frac{\mathbb{E}_0^l \left[\int_0^{\tau_m \wedge \tau_F} e^{-rt} dt \right]}{\mathbb{E}_0^l \left[\int_0^{\tau_F} e^{-rt} dt \right]} \left(D_l(\infty) - q \right), \quad \forall m > \underline{m}_h.$$

[9] For tractability, I assume there are only two types of firms. Having more than two types of firms would imply that I have to keep track of the whole distribution of beliefs. Although this assumption is restrictive, the driving forces described in this setup would still exist in a richer setup.

The left-hand side, $K_h(m)$, is the expected discounted cash flow shareholders of an unlevered high-quality firm would receive in the time debt with maturity m is outstanding, which is a lower bound on the debt proceeds minus the issuance cost. The right-hand side is a low-quality firm's proceeds from debt issuance incase it reveals its type and issues perpetual debt, $D_l(\infty) - q$, scaled by the expected discounted fraction of the firm's life debt with maturity m is outstanding,

$$\frac{\mathbb{E}_0^l\left[\int_0^{\tau_m \wedge \tau_F} e^{-rt} dt\right]}{\mathbb{E}_0^l\left[\int_0^{\tau_F} e^{-rt} dt\right]}.$$

This assumption is sufficient but not necessary for existence of the equilibrium I study.

Assume that the firm is unlevered at time zero when the issuance game starts. Then issuance can happen at any time $t \geq 0$. At each instant dt, competitive creditors post a private offer (W_t, M_t). Creditors are willing to pay W_t for debt with maturity M_t and coupon $c = 1$. As in He and Milbradt (2016), I assume that the coupon c is fixed and focus on the firm's timing of debt issuance and debt maturity choice.[10] Shareholders decide whether to accept the offer and issue debt or to reject the offer.[11,12] The offer W_t can be interpreted as the highest offer for debt with maturity M_t made by competitive creditors. After acceptance, the firm issues debt with maturity M_t and receives W_t. After rejecting the offer, creditors observe the news dI_t, which I discuss in more detail in the next subsection, and learn whether the firm survived $t < \tau_F$. If the firm survives, then we move to the next instant, and the process repeats itself until shareholders decide to accept the offer or the firm fails. Let τ^* be the time at which shareholders accept the offer. The top part of Fig. 2 graphically describes the issuance game.

This setup generates an issuance game in which creditors need to post an offer and shareholders need to decide whether to accept the offer and issue debt or reject the offer. Before turning to creditors' and shareholders' strategy space, I describe creditors' information set $\mathcal{F}_t$, which contains all information after the events at time t have realized, and how

[10] The trade-offs described here should still exist in a richer setup where leverage is also endogenized. In Geelen (2016), I study firms' joint choice of leverage and debt maturity without commitment in a dynamic setting with perfect information.

[11] The offers are shareholders' private information to avoid signaling, see Noldeke and Damme (1990) and Hörner and Vieille (2009).

[12] Later, I discuss why there is no difference between creditors making an offer for a single debt maturity M_t or creditors making an offer for every debt maturity $m \in \mathbb{R}_+$.

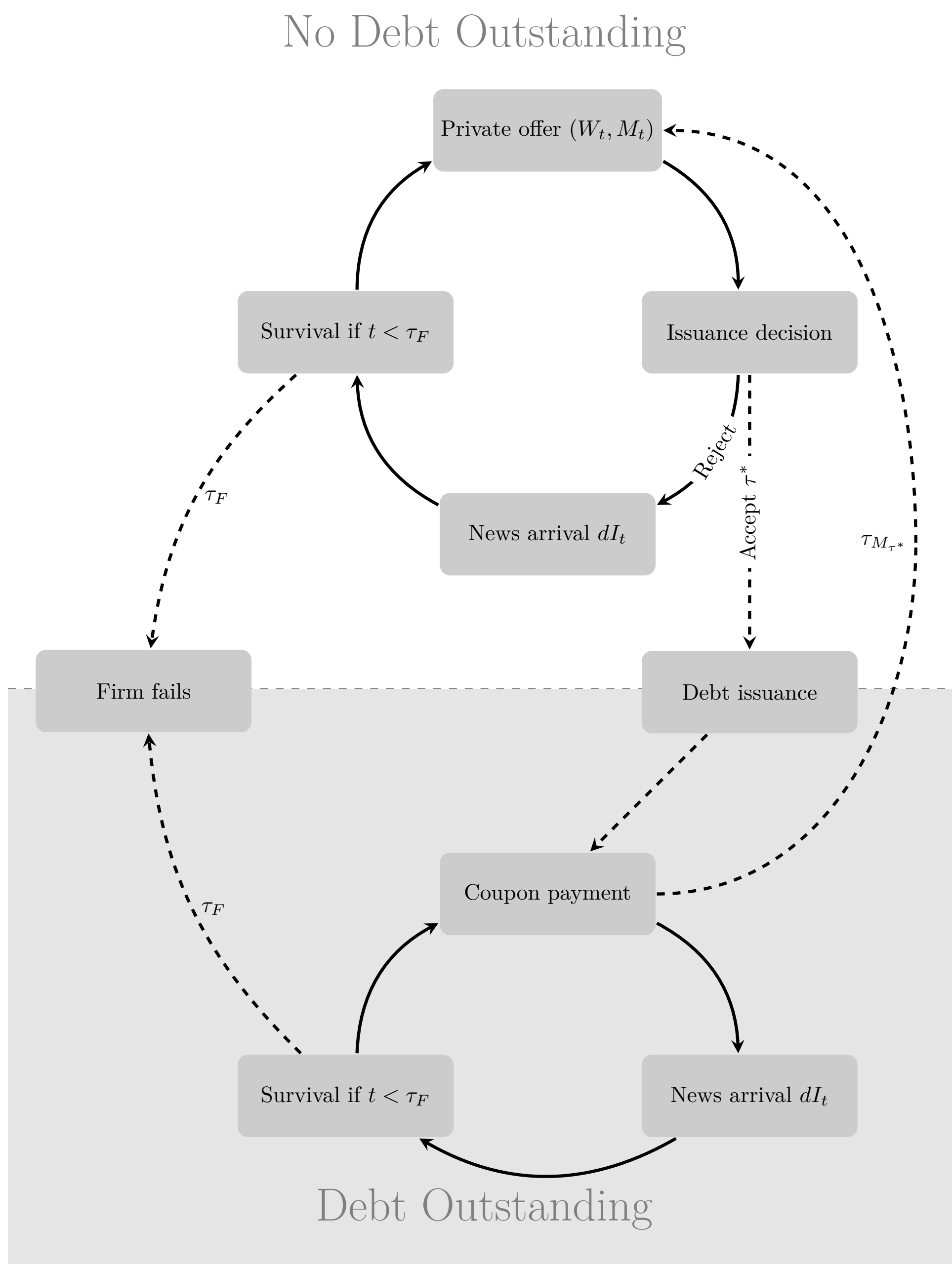


FIGURE 2: **Description of the repeated debt issuance game.** The issuance game ends when the firm fails at τ_F. The firm issues debt at τ^*. This debt has a maturity M_{τ^*} and matures $\tau_{M_{\tau^*}}$ time later.

this relates to creditors offers. This filtration contains the following information: the news creditors received I_t, whether the firm survived $t < \tau_F$, when the firm issued debt and when this debt matured, and randomization devices.[13] In my model, the randomization devices are

[13]Formally, the randomization devices are only part of shareholders filtration and not of creditors filtration. To keep notation simple, I just work with one filtration $(\mathcal{F}_t)_{t\geq 0}$.

draws from an exponential distribution. The offer creditors make at time t can only depend on information in $\mathcal{F}_{t^-}$, where $t^- = \lim_{s \uparrow t} s$, because creditors do not yet know what happens at time t. The stochastic processes $(W, M) = (W_t, M_t)_{t \geq 0}$ describe creditors offer strategy and are adapted to the filtration $(\mathcal{F}_{t^-})_{t \geq 0}$. Shareholders of a quality θ firm's issuance strategy is a stopping time τ_θ of $(\mathcal{F}_t)_{t \geq 0}$.

After the firm has issued debt for the first time at τ^*, shareholders start making coupon payments to creditors. Furthermore, creditors continue to learn about the firm's quality from the news and its survival. After the debt has matured at $\tau^* + \tau_{M_{\tau^*}}$, assuming the firm survived, the issuance game described above repeats itself. This implies that from time $\tau^* + \tau_{M_{\tau^*}}$ onwards creditors need to make offers and shareholders need to decide whether to accept or reject these offers. Fig. 2 describes the repeated debt issuance game. It contains the single round debt issuance game with the possibility that after the debt matures the firm can issue debt again. The repeated issuance game ends when the firm fails at τ_F. Because I study stationary equilibria of the issuance game, the strategy space for creditors and shareholders in the single issuance game is also their strategy space in the repeated issuance game.[14]

C. *Learning*

Creditors learn about the firm's quality from news, its survival, and its debt issuance behavior. This learning causes the asymmetric information to decrease over time.

In my model, the exogenous news process I_t is an aggregation of news revealed about the firm's prospects. In practice, this news could be analysts' recommendations, news released about the firm or its industry by the Financial Times or Bloomberg, news about production delays, news about clinical trials' results, etc. News reveals itself to creditors and allows them to update their beliefs about the firm's quality. This news process I_t evolves according to

$$dI_t = \mathbb{1}_{\{\theta = h\}} dt + \frac{1}{\phi} dB_t,$$

where B_t is a standard Brownian motion and ϕ measures the news quality. The drift of this news process is informative about the firm's quality. The higher the news quality ϕ the

[14] The one-shot deviation principle ensures that the stationary equilibrium I construct is a subgame perfect equilibrium. (See also footnote 16.)

more informative the news process is and therefore the faster creditors learn about the firm's quality. One can think of Brownian news as gradual revelation of information about the firm's quality.

Creditors use Bayes' rule to update their beliefs about the firm being high quality:

$$P_t = \mathbb{P}(\theta = h | \mathcal{F}_t, t < \tau_F).$$

To write down creditors' beliefs about the firm's quality, it is necessary to know the probability that a firm of quality θ survives until time t. The failure time of the firm has an exponential distribution with intensity λ_θ. For a firm of quality θ, the news I_t is normally distributed with mean $\mathbb{1}_{\{\theta=h\}}t$ and variance $\phi^{-2}t$. Define $f_t^\theta(I)$ as the cumulative distribution function of a normal random variable with mean $\mathbb{1}_{\{\theta=h\}}t$ and variance $\phi^{-2}t$. Creditors' initial prior that the firm is high quality is P_{0^-}. The time $g_{is}(t)$ is the last time before time t the firm issued debt (or time zero). The time $g_{mat}(t)$ is the last time before time t the firm's debt matured (or time zero).

Because the model is a repeated issuance game, there are different cases:

1. *The firm is unlevered*: Creditors' beliefs evolve according to

$$\begin{aligned}
A_t^h &= \underbrace{P_{g_{mat}^-(t)}}_{prior} \underbrace{f_{t-g_{mat}(t)}^h \left(I_t - I_{g_{mat}(t)}\right)}_{news} \underbrace{e^{-\lambda_h(t-g_{mat}(t))}}_{survival} \underbrace{\mathbb{P}_t(\tau_h > t)}_{issuance}, \\
A_t^l &= \left(1 - P_{g_{mat}^-(t)}\right) f_{t-g_{mat}(t)}^l \left(I_t - I_{g_{mat}(t)}\right) e^{-\lambda_l(t-g_{mat}(t))} \mathbb{P}_t(\tau_l > t), \\
P_t &= \frac{A_t^h}{A_t^h + A_t^l}.
\end{aligned} \tag{1}$$

   Creditors update their beliefs based on news, the firm's survival, and absence of debt issuance. It is important to realize that the firm's issuance strategy τ_θ matters for creditors' beliefs. For example, if a low-quality firm issues debt before time t, $\mathbb{P}_t(\tau_l > t) = 0$, while a high-quality firm postpones issuance until after time t, $\mathbb{P}_t(\tau_h > t) = 1$, then upon observing the firm postponing debt issuance until time t, creditors infer that the firm is high quality and update their beliefs $P_t = 1$.

2. *The firm issues debt*: Observing debt issuance at time t is also an informative signal,

and therefore creditors update their beliefs:

$$P_t = \frac{P_{t^-} \mathbb{P}_t(\tau_h = t)}{P_{t^-} \mathbb{P}_t(\tau_h = t) + (1 - P_{t^-})\, \mathbb{P}_t(\tau_l = t)}. \tag{2}$$

For example, if at time t only a low-quality firm would issue debt and creditors observe debt issuance then beliefs jump to zero.

3. *The firm has debt outstanding*: Creditors' beliefs evolve according to

$$P_t = \frac{P_{g_{is}^-(t)} f^h_{t-g_{is}(t)} \left(I_t - I_{g_{is}(t)}\right) e^{-\lambda_h (t - g_{is}(t))}}{P_{g_{is}^-(t)} f^h_{t-g_{is}(t)} \left(I_t - I_{g_{is}(t)}\right) e^{-\lambda_h (t - g_{is}(t))} + \left(1 - P_{g_{is}^-(t)}\right) f^l_{t-g_{is}(t)} \left(I_t - I_{g_{is}(t)}\right) e^{-\lambda_l (t - g_{is}(t))}}. \tag{3}$$

Creditors update their beliefs based on news and the firm's survival.

These three cases show that creditors always learn from news and the firm's survival. Furthermore, when the firm is unlevered creditors also learn from both the firm postponing debt issuance and the firm issuing debt.

III. Equilibrium

In this section, I describe the equilibrium concept and construct a "natural" equilibrium. I focus on stationary equilibria in which the state can be summarized by creditors' beliefs p.[15] This implies that creditors' offer (W_t, M_t) is a function of their beliefs,

$$(W_t, M_t) = (W(P_{t^-}), M(P_{t^-})).$$

[15] I call p the state variable and P_t the belief process. The expectation $\mathbb{E}^\theta_p$ is conditional on θ and creditors' beliefs at time zero before they make their first offer $P_{0^-} = p$. I define the expectations operator $\mathbb{E}_p$ in a similar way.

The equity value of an unlevered firm conditional on offers (W, M) is

$$E_\theta(p) = \sup_\tau \left\{ \mathbb{E}_p^\theta \left[\int_0^{\tau \wedge \tau_F} (1-\pi) ds \right] \right. \tag{4}$$

$$\left. + \mathbb{E}_p^\theta \left[\mathbb{1}_{\{\tau < \tau_F\}} e^{-r\tau} \left\{ W(P_{\tau^-}) + \mathbb{1}_{\left\{\tau + \tau_{M(P_{\tau^-})} < \tau_F\right\}} e^{-r\tau_{M(P_{\tau^-})}} E_\theta \left(P_{\tau + \tau^-_{M(P_{\tau^-})}} \right) - q \right\} \right] \right\}.$$

This equation shows that the firm generates an after-tax dividend of $1-\pi$ until it issues debt at time τ or fails at time τ_F. If the firm issues debt, equity value is the debt proceeds $W(P_{\tau^-})$ plus the discounted equity value once the debt matures $\mathbb{1}_{\{\tau+\tau_{M(P_{\tau^-})}<\tau_F\}} e^{-r\tau_{M(P_{\tau^-})}} E_\theta(P_{\tau+\tau^-_{M(P_{\tau^-})}})$ minus the issuance cost q. If the firm fails, equity value drops to zero. The equity value today $E_\theta(p)$ depends on the equity value in the future $E_\theta(P_{\tau+\tau^-_{M(P_\tau^-)}})$. The reason is that after the debt matures, at $\tau + \tau_{M(P_{\tau^-})}$, shareholders own all the firm's operating income again and have the possibility to issue debt again. The equity value when the firm has debt outstanding is given by

$$\mathbb{E}_p^\theta \left[\mathbb{1}_{\{\tau_m < \tau_F\}} e^{-r\tau_m} E_\theta \left(P_{\tau_m^-} \right) \right].$$

The firm's operating income is used to pay creditors until the maturity date τ_m. If the debt matures before the firm fails, shareholders own the unlevered equity value $E_\theta\left(P_{\tau_m^-}\right)$. If the firm fails before the debt matures, equity value drops to zero. Shareholders maximize the equity value by choosing the optimal issuance strategy. Optimality of the issuance strategy implies that τ_θ solves the optimization problem on the right-hand side of equation (4).

The equilibrium definition is

Definition 1. *An equilibrium to the dynamic issuance game is a quintuple* $(\tau_l, \tau_h, W, M, P)$ *such that:*

1. ***Issuance Optimality****. Given offers* (W, M)*,* τ_θ *solves a quality* θ *firm's issuance problem in equation (4).*[16]

[16] I only study stationary equilibria. Therefore, the fact that the issuance strategy solves equation (4) ensures that no profitable one-shot deviation exists for shareholders. This ensures that the solution is a subgame perfect equilibrium since creditors are competitive.

2. ***Belief Consistency***. *Creditors correctly update their beliefs so that the belief process* P_t *follows equations* (1)-(3).

3. ***Zero Profit***. *If there exists a probability that the firm issues debt,* $\tau^* = t$*, then creditors' offer is competitive,*

$$W(P_{t^-}) = \mathbb{E}_{t^-}\left[D_\theta(M(P_{t^-}))|\tau^* = t\right].$$

4. ***No Deals***. *There does not exist an offer that is accepted by shareholders and leads to a positive expected profit for creditors. For a high-quality firm, for any beliefs p, we have*

$$E_h(p) \geq \sup_m \left\{\mathbb{E}_p\left[D_\theta(m)\right] + \mathbb{E}^h_p\left[\mathbb{1}_{\{\tau_m<\tau_F\}}e^{-r\tau_m}E_h\left(P_{\tau_m^-}\right)\right] - q\right\}.$$

Similarly, for a low-quality firm, for any beliefs p, we have

$$E_l(p) \geq \sup_m \left\{D_l(m) + \mathbb{E}^l_p\left[\mathbb{1}_{\{\tau_m<\tau_F\}}e^{-r\tau_m}E_l(0)\right] - q\right\}.$$

The no deals condition ensures that there are no profitable offers for creditors that would be accepted by shareholders. This condition follows from the fact that creditors are competitive. Without the no deals condition, creditors always offering zero and shareholders always refusing creditors' offer would be an equilibrium. In this case, a creditor could deviate and offer $w \in (K_l(\infty), D_l(\infty))$ for perpetual debt. This offer would be accepted by shareholders of a low-quality firm because $w > K_l(\infty)$ and would be profitable for the creditor because $w < D_l(\infty)$. The no deals condition rules out the existence of such deals. Because of Assumption 1, shareholders of a high-quality firm would never issue debt at a low-quality firm's valuation $D_l(m)$. Therefore, if a firm issues debt at this price creditors infer that the firm must be low quality. This result leads to the no deals condition for a low-quality firm, which would reveal its type by issuing debt at a price $D_l(m)$.

The fact that creditors only make an offer for a single debt maturity raises the question whether any equilibrium would break down when creditors could make offers for multiple debt maturities, as happens in the insurance market with asymmetric information of Rothschild and Stiglitz (1976) and Wilson (1977). Assuming the firm issues debt, this is not the case because any equilibrium is also a Wilson equilibrium, see Wilson (1977) and Rothschild and Stiglitz (1976), in a model where creditors could make an offer for any debt maturity $m \in \mathbb{R}_+$.[17] Given the no deals conditions, for shareholders of a high-quality firm to have a (strict) incentive to deviate to a new offer $(m', W(m'))$ creditors must offer atleast $W(m') > \mathbb{E}_p[D_\theta(m')]$. If high-quality firms would deviate, then the equilibrium offer $(m, W(m))$ would be withdrawn since its no longer profitable. Shareholders of a low-quality firm would then pool with a high-quality firm and issue debt with maturity m' but in this case creditors who offer $(m', W(m'))$ would make a loss, which rules out m' being offered. Any new offer accepted by only shareholders of a low-quality firm would reveal the firm's quality. These off-equilibrium beliefs in combination with the no deals conditions ensure that no low-quality firm would accept another offer for a different debt maturity m', which shows that conditional on debt issuance any equilbrium that satisfies the definition given above is a Wilson equilibrium.

Define by $\Phi(p, m)$ the creditors' expected debt value conditional on beliefs p:

$$\Phi(p, m) = \mathbb{E}_p\left[D_\theta(m)\right] = pD_h(m) + (1-p)D_l(m).$$

A natural conjecture for an equilibrium of the issuance game is a threshold equilibrium (α, β): for beliefs $p \in (\alpha, \beta)$ the firm postpones debt issuance, above β both types of firm issue debt, and below α only a low-quality firm issues debt. There exists a threshold β such that for beliefs above β, shareholders of a high-quality firm prefer to issue debt because the tax benefits outweigh the increase in the debt price due to learning. Shareholders of a low-quality firm want to sell overpriced debt. Therefore, above β a low-quality firm mimics a high-quality firm and also issues debt. Furthermore, debt maturity is increasing in beliefs above β because the higher creditors' beliefs, the less important adverse selection becomes relative to the issuance costs. For decreasing beliefs that are below β, the chance of a low-quality firm mimicking a high-quality firm decreases because the probability that it fails before reaching β is higher. At some point, shareholders of a low-quality firm are indifferent

[17]The assumption that creditors issue debt can be relaxed in two ways. One way is that firms first commit to issuing debt before creditors make offers. Another way is that off-equilibrium beliefs are set such that if a high-quality firm is expected to issue debt then a firm's refusal to issue debt would indicate its low quality.

between issuing debt and revealing their type and postponing debt issuance and mimicking a high-quality firm. This indifference pins down α. Below α, a low-quality firm mixes between issuance and postponing issuance so that its shareholders remain indifferent between the two.

A low-quality firm that issues debt at or below α reveals its type. Therefore, adverse selection no longer plays a role, and shareholders of a low-quality firm issue perpetual debt to minimize issuance costs. Creditors beliefs are reflected at α upon the firm postponing debt issuance. In this case, below α the equity value of a low-quality firm that postpones debt issuance is equal to the equity value at α. By construction of α, the equity value at this point is equal to the equity value of a low-quality firm that reveals its type and issues debt.

$$E_l(P_t) = E_l(P_{t^+}) = E_l(\alpha) = D_l(\infty) - q, \quad \forall P_t \leq \alpha.$$

At or below α, the issuance strategy for shareholders of a low-quality firm is constructed so that beliefs are reflected at α upon the firm postponing debt issuance.

The next step is to formalize this natural conjecture for an equilibrium. Given a constant $\alpha \in [0, 1]$, a set $\mathcal{B} \subseteq [\alpha, 1]$, and a function $\mathcal{M} : [0, 1] \to \mathbb{R}_+$, I first define the belief process and strategy profile $\mathcal{S}(\alpha, \mathcal{B}, \mathcal{M})$. The set $\mathcal{B}$ generalizes shareholders issuance strategy for $p \geq \alpha$. In all the numerical examples that follow, the issuance region for $p \geq \alpha$ is of threshold type $\mathcal{B} = [\beta, 1]$. Furthermore, I can prove that in any equilibrium for every belief $p \in \mathcal{B}$ with $p < 1$ there exists a belief $p' \in (p, 1)$ that is also in $\mathcal{B}$.

Definition 2 (Evolution of Beliefs)**.** *For a belief process and strategy profile $\mathcal{S}(\alpha, \mathcal{B}, \mathcal{M})$, creditors' beliefs evolve according to:*

1. *If the firm is unlevered, beliefs follow equation (1) with the issuance strategy of a low-quality firm chosen such that beliefs are reflected at α upon the firm postponing debt issuance in that*

$$P_t \geq \alpha.$$

The probability that a low-quality firm would have issued debt is $\mathbb{P}_t(\tau_l \leq t) = 1 - e^{-L_t^\alpha}$

with

$$L_t^\alpha = \max\left(\log\left(\frac{\alpha}{1-\alpha}\right) - \inf_{s\in[g_{mat}(t),t]}\log\left(\frac{P_{g^-_{mat}(t)} f^h_{s-g_{mat}(t)}\left(I_s - I_{g_{mat}(t)}\right)e^{-\lambda_h(s-g_{mat}(t))}}{\left(1-P_{g^-_{mat}(t)}\right) f^l_{s-g_{mat}(t)}\left(I_s - I_{g_{mat}(t)}\right)e^{-\lambda_l(s-g_{mat}(t))}}\right), 0\right).$$

2. *When the firm issues debt, creditors' beliefs follow equation* (2)*.*

3. *If the firm is levered, creditors' beliefs follow equation* (3)*.*

When the firm is unlevered, beliefs evolve due to the firm's survival. Furthermore, a low-quality firm's issuance strategy ensures that beliefs are reflected at α upon the firm postponing debt issuance. This reflection, in combination with equation (1) and the fact that $\mathbb{P}(\tau_h > t) = 1$ for $P_t \notin \mathcal{B}$, implies that

$$\frac{1-\alpha}{\alpha}\frac{P_{g^-_{mat}(t)} f^h_{t-g_{mat}(t)}\left(I_t - I_{g_{mat}(t)}\right)e^{-\lambda_h(t-g_{mat}(t))}}{\left(1-P_{g^-_{mat}(t)}\right) f^l_{t-g_{mat}(t)}\left(I_t - I_{g_{mat}(t)}\right)e^{-\lambda_l(t-g_{mat}(t))}} - \mathbb{P}_t(\tau_l > t) \geq 0. \tag{5}$$

The first term is the likelihood ratio of beliefs that only evolves due to news and the firm's survival. Reflection of beliefs at α implies that the probability that shareholders of a low-quality firm postpone issuance $\mathbb{P}_t(\tau_l > t)$ is the largest probability that satisfies equation (5):

$$\begin{aligned}\mathbb{P}_t(\tau_l > t) &= \min\left(\inf_{s\in[g_{mat}(t),t]}\frac{1-\alpha}{\alpha}\frac{P_{g^-_{mat}(t)} f^h_{s-g_{mat}(t)}\left(I_s - I_{g_{mat}(t)}\right)e^{-\lambda_h(s-g_{mat}(t))}}{\left(1-P_{g^-_{mat}(t)}\right) f^l_{s-g_{mat}(t)}\left(I_s - I_{g_{mat}(t)}\right)e^{-\lambda_l(s-g_{mat}(t))}}, 1\right)\\ &= e^{-\max\left(\log\left(\frac{\alpha}{1-\alpha}\right)-\inf_{s\in[g_{mat}(t),t]}\log\left(\frac{P_{g^-_{mat}(t)} f^h_{s-g_{mat}(t)}\left(I_s-I_{g_{mat}(t)}\right)e^{-\lambda_h(s-g_{mat}(t))}}{\left(1-P_{g^-_{mat}(t)}\right) f^l_{s-g_{mat}(t)}\left(I_s-I_{g_{mat}(t)}\right)e^{-\lambda_l(s-g_{mat}(t))}}\right),0\right)}\\ &= e^{-L_t^\alpha}.\end{aligned}$$

A smaller probability of a low-quality firm postponing debt issuance causes beliefs to be reflected at a level above α. A larger probability of a low-quality firm postponing debt issuance causes beliefs to be reflected at a level below α. For $P_t \notin \mathcal{B}$, the stopping time $\inf\{t \geq g_{mat}(t) | L_t^\alpha \geq \xi\}$ with $\xi \sim \exp(1)$ has the same distribution as τ_l.

Definition 3 (Issuance Strategy)**.** *For a belief process and strategy profile $\mathcal{S}(\alpha, \mathcal{B}, \mathcal{M})$, shareholders issuance strategy is:*

1. *A high-quality unlevered firm issues debt when $P_t \in \mathcal{B}$,*

$$\tau_h = \inf\{t \geq g_{mat}(t) | P_t \in \mathcal{B}\},$$

   and the debt issued has maturity $\mathcal{M}(P_{\tau_h})$. A high-quality levered firm already reached its debt capacity and therefore does not issue any debt.

2. *A low-quality unlevered firm issues debt when $P_t \in \mathcal{B}$ and mixes between issuance and postponing issuance when $P_t \leq \alpha$. For $P_t \leq \alpha$, the issuance probability is set such that beliefs are reflected at α upon the firm postponing debt issuance.*

$$\tau_l = \inf\{t \geq g_{mat}(t) | P_t \in \mathcal{B} \text{ or } L_t^{\alpha} \geq \xi\},$$

   with $\xi \sim \exp(1)$. The debt issued has maturity $\mathcal{M}(P_{\tau_l})$ when $P_{\tau_l} \in \mathcal{B}$ and infinite maturity otherwise. A low-quality levered firm already reached its debt capacity and therefore does not issue any debt.

Both a high and low-quality firm issue debt when beliefs are inside $\mathcal{B}$. Therefore, creditors offer debt with maturity $\mathcal{M}(p)$ at a price $\Phi(p, \mathcal{M}(p))$ when beliefs are inside $\mathcal{B}$. Outside $\mathcal{B}$, either no firm issues debt or only a low-quality firm issues debt. Therefore, creditors offer perpetual debt at a low-quality firm's valuation $D_l(\infty)$ when beliefs are outside $\mathcal{B}$.

Definition 4 (Offer Strategy)**.** *For a belief process and strategy profile $\mathcal{S}(\alpha, \mathcal{B}, \mathcal{M})$, creditors offer*

$$W(p) = \begin{cases} \Phi(p, \mathcal{M}(p)), & \text{if } p \in \mathcal{B}, \\ D_l(\infty), & \text{if } p \notin \mathcal{B}, \end{cases}$$

$$M(p) = \begin{cases} \mathcal{M}(p), & \text{if } p \in \mathcal{B}, \\ \infty, & \text{if } p \notin \mathcal{B}. \end{cases}$$

To get more intuition on how the belief process and strategy profile $\mathcal{S}(\alpha, \mathcal{B}, \mathcal{M})$ works, Fig. 3 plots two paths of the belief process assuming that $\mathcal{B} = [\beta, 1]$. Because creditors receive Brownian news their beliefs difuse. Fig. 3(a) shows a path of the belief process for a high or low-quality firm. First, beliefs drift downwards and are reflected at α because the firm postpones debt issuance, which is a positive signal to creditors. Then beliefs drift upwards, and when they reach β the firm issues debt with maturity $\mathcal{M}(\beta)$. This debt matures at $\tau_1^* + \tau_{\mathcal{M}(\beta)}$, when beliefs are below α. Because the firm postpones debt issuance beliefs jump up to α. Beliefs then drift upwards to β where the firm issues debt for the second time with maturity $\mathcal{M}(\beta)$. This debt matures at $\tau_2^* + \tau_{\mathcal{M}(\beta)}$ and because beliefs are inside $\mathcal{B}$ the firm replaces the maturing debt with new debt that has maturity $\mathcal{M}(P_{\tau_2^* + \tau_{\mathcal{M}(\beta)}}) \neq \mathcal{M}(\beta)$. Fig. 3(b) shows a path of the belief process for a low-quality firm. Beliefs drift downwards. When beliefs reach α for the first time, the firm postpones debt issuance, which is a positive signal to creditors and therefore causes beliefs to be reflected at α. When beliefs reach α for the second time, the firm issues perpetual debt and thus reveals its type. Finally, the firm fails at τ_F.

To prove the existence of an equilibrium, I need to specify off-equilibrium beliefs in case shareholders refuse an offer when $p \in \mathcal{B}$.[18] In the proof that constructs an equilibrium α, $\mathcal{B}$, and $\mathcal{M}$, the set $\mathcal{B}$ is determined as the region where shareholders of a high-quality firm issue debt assuming creditors offer $\Phi(p, m)$ for every debt maturity $m \in \mathbb{R}_+$, and beliefs P_t evolve due to the firm's survival and are reflected at α that is, we have

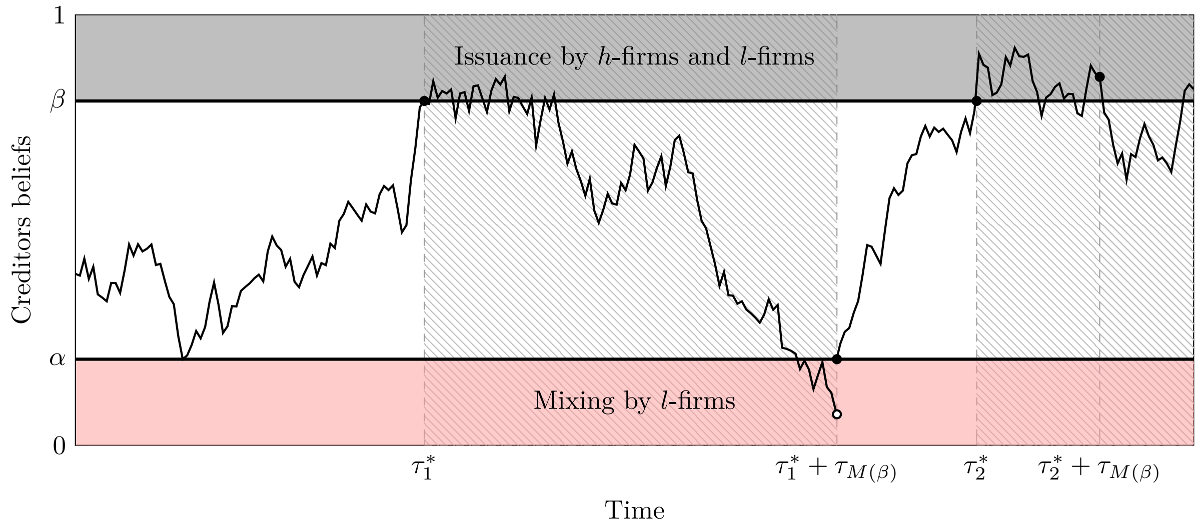
$$\mathcal{B} = \left\{ p \geq \alpha \middle| E_h(p) = \sup_{m \geq 0} \underbrace{\Phi(p, m) - q}_{\textit{net debt proceeds}} + \underbrace{\mathbb{E}_p^h \left[\mathbb{1}_{\{\tau_m < \tau_F\}} e^{-r\tau_m} E_h(P_{\tau_m^-}) \right]}_{\textit{levered equity value}} \right\},$$

and for $p \in \mathcal{B}$ the optimal debt maturity is

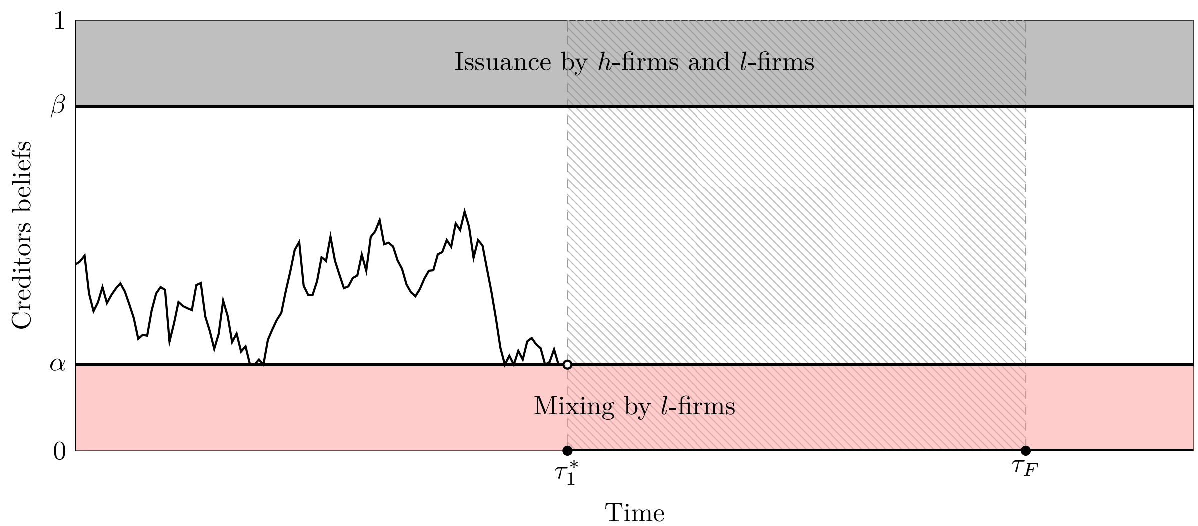
$$\mathcal{M}(p) \in \arg\sup_{m \geq 0} \left\{ \Phi(p, m) - q + \mathbb{E}_p^h \left[\mathbb{1}_{\{\tau_m < \tau_F\}} e^{-r\tau_m} E_h(P_{\tau_m^-}) \right] \right\}.$$

Off-equilibrium threats do not drive the construction of $\mathcal{B}$ and shareholders of a high-quality

[18] The difficulty lies in showing that shareholders of a low-quality firm have no incentive to delay debt issuance because a priori their value function is not continuous in p since the optimal debt maturity is not necessarily continuous in p.



(a) Path of the belief process for a high or low-quality firm



(b) Path of the belief process for a low-quality firm

FIGURE 3: **Two paths of the belief process P_t for the belief process and strategy profile $\mathcal{S}(\alpha, \mathcal{B}, \mathcal{M})$.** The issuance region $\mathcal{B} = [\beta^*, 1]$. In Fig. 3(a), the firm is either high or low-quality and issues debt three times. In Fig. 3(b), the firm is low quality and issues debt once and fails later.

firm always want to issue debt for $p \in \mathcal{B}$. Given this construction of $\mathcal{B}$, I specify my off-equilibrium beliefs as

Assumption 2. *For $p \in \mathcal{B}$, if shareholders refuse an offer then creditors conjecture the firm must be low quality.*

In all the numerical examples that follow, these off-equilibrium beliefs are void and could be replaced by creditors' beliefs not responding to the firm refusing the offer.

To make the analysis that follows transparent, I shut down the Brownian news process and show that it does not driven my results.

Assumption 3. *The Brownian news process is uninformative, $\phi \to 0$.*

Section V shows that the results I find are robust to incorporating a Brownian news process into my model.

The following theorem establishes this section's main result:

Theorem 1. *There exists a constant α^*, a set $\mathcal{B}^*$, and a function $\mathcal{M}^*$ such that $\mathcal{S}(\alpha^*, \mathcal{B}^*, \mathcal{M}^*)$ is an equilibrium. Furthermore, shareholders postpone debt issuance in some non-empty region above α^*.*

IV. Model Implications

In this section, I study the model's implications. I first discuss firms' optimal debt maturity choice and the existence of zero-leverage firms. I then perform a numerical comparative statics analysis to obtain cross-sectional empirical predictions. Finally, I look at the effects firms' ability to choose their debt maturity on the equilibrium outcomes.

A. Optimal Debt Maturity

When determining the optimal debt maturity, shareholders trade off the benefits of shorter maturity debt that is the lower adverse selection and the possibility to issue debt with lower underpricing when the debt matures, against the frequency with which they incur the issuance cost. This suggests that when uncertainty about the firm's quality becomes smaller, the firm should issue longer maturity debt because adverse selection becomes less important relative to the issuance costs. The following proposition formalizes this intuition.

Proposition 2. *For $p \geq \alpha^*$, there exists a strictly increasing lower bound on the optimal debt maturity $\mathbb{M}(p)$,*

$$\mathcal{M}(p) \geq \mathbb{M}(p).$$

Furthermore, when creditors become more certain that the firm is high-quality

$$\lim_{p \to 1} \mathbb{M}(p) = \infty.$$

The proposition tells us that there exists an increasing lower bound on the optimal debt maturity $\mathbb{M}(p)$ that converges to perpetual debt as creditors become more certain that the firm is high quality.

I numerically solve the model to obtain the optimal issuance strategy. The base case parameters are given in Table 1. I set the model parameters as follows. The interest rate is 4.2%, which is similar to Morellec et al. (2012)'s estimate. The tax benefits to debt are equal to 15%, the same estimate as Hugonnier et al. (2015) use, which is based on Graham (1996). I set issuance cost q equal to 1.09% of a high-quality firm's perpetual debt value given the base case parameters, which is Altinkilic and Hansen (2000) estimate of the average debt issuance cost. Finally, I set the failure intensities such that a high-quality firm is expected to generate operating income for 50 years and a low-quality firm for 10 years.

TABLE 1: **The base case parameters.**

Parameter	Symbol	Value
Interest rate	r	4.2%
Tax benefits	π	15%
Issuance cost	q	$1.09\% * D_h^{base}(\infty)$
Failure intensity h-firm	λ_h	0.02
Failure intensity l-firm	λ_l	0.1

The upper issuance threshold is 0.57, and the lower issuance threshold is 0.48. Fig. 4 plots the issuance regions and optimal debt maturity offered by creditors. For $p \geq \beta^*$, shareholders optimally issue debt and the optimal debt maturity is increasing in beliefs, which is consistent with the lower bound on the optimal debt maturity found in Proposition 2. For $p < \alpha^*$, only shareholders of a low-quality firm issue debt, and this debt has perpetual maturity.

These results illustrate the life-cycle theory of debt maturity in which younger firms issue shorter maturity debt, while mature firms issue longer maturity debt. This life-cycle theory of debt maturity is consistent with Barclay and Smith (1995), Berger et al. (2005), and Custódio et al. (2013), who show that firms that face more asymmetric information issue shorter maturity debt, and Fig. 1(a), which shows that firm age and debt maturity are positively correlated.

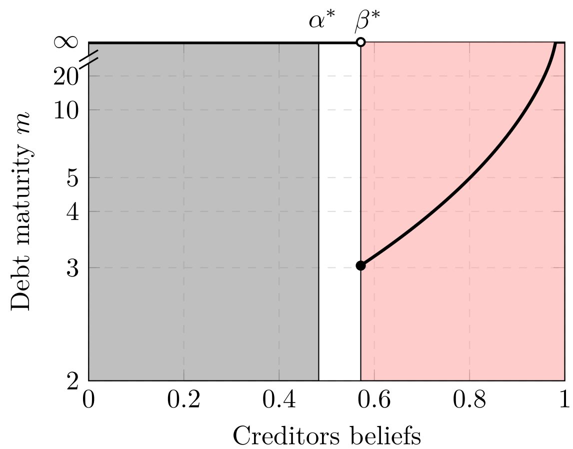


FIGURE 4: **The base case optimal issuance strategy and debt maturity.** The solid line depicts the debt maturity offered by creditors.

B. *Zero-Leverage Firms*

Theorem 1 shows that there exists a non-empty region above α^* where the firm postpones debt issuance. This region is

$$\mathcal{R} = (\alpha^*, 1] \cap (\mathcal{B}^*)^c,$$

where $(\mathcal{B}^*)^c$ the complement of $\mathcal{B}^*$.

Proposition 3. *For beliefs $p \in \mathcal{R}$, the firm postpones debt issuance and has zero leverage.*

The firm has zero leverage even though under perfect information issuing debt is positive NPV,

$$D_\theta(\infty) > K_\theta(\infty) - q.$$

Shareholders of a high-quality firm postpone debt issuance to increase the debt price, which happens because creditors learn about the firm's quality. Shareholders of a low-quality firm postpone debt issuance in the hope of selling overpriced debt in the future, which happens because they can mimic a high-quality firm.

As soon as beliefs p reach $\mathcal{B}^*$, the firm issues debt. When beliefs p are at or below α^*, the firm, if it is low quality, mixes between issuance and postponing issuance. In equilibrium, a firm is therefore expected to issue debt, which has consequences for its value as shown in the next proposition.

Proposition 4. *For a zero-leverage firm ($p \in \mathcal{R}$), there is a positive probability that the firm issues debt in the future in that*

$$\mathbb{P}_p^{\theta}(\tau_\theta < \tau_F) > 0.$$

As a result, firm value, equity value for a zero-leverage firm, is larger than unlevered firm value in that

$$\mathbb{E}_p\left[E_\theta(p)\right] > \mathbb{E}_p\left[\int_0^{\tau_F} e^{-rt}(1-\pi)dt\right].$$

This proposition is consistent with Korteweg (2010)'s finding that (dividend paying) zero-leverage firms have net benefits to debt worth 2.5% of firm value and thus are expected to issue debt in the future. Firm value reflects this future change in debt policy.

Over time, creditors update their beliefs, which decreases information asymmetries, and at some point the firm issues debt. Consistent with this finding, Fig. 1(b) shows empirically that the fraction of zero-leverage firms is decreasing in firm age. Also consistent with this finding, Fig. 5 shows empirically that the probability of having zero leverage in x years conditional on having zero leverage today is decreasing in the number of years x.

Finally, before debt issuance payouts to shareholders are $(1-\pi) > 0$, which proves the following proposition.

Proposition 5. *A zero-leverage firm ($p \in \mathcal{R}$) makes payouts to its shareholders.*

The reason that shareholders prefer to leave tax benefits on the table and pay out earnings is that shareholders hope to sell debt in the future at a higher price. My model is therefore consistent with the existence of zero-leverage firms that make payouts, see Fig. 6.[19]

C. Comparative Statics Analysis

This subsection performs a numerical comparative statics analysis with respect to all the model's parameters. Fig. 7 shows the results of the comparative statics analysis. The

[19] To make the analysis transparent, I do not incorporate investment opportunities into the model. Incorporating investment opportunities may lead some of these firms to retain earnings.

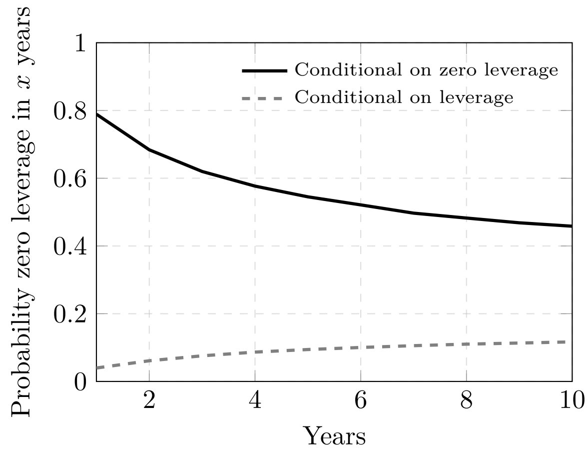


FIGURE 5: **Transition probabilities of zero-leverage firms.** The figure shows the transition probabilities of having zero leverage (DLTT+DLC=0). All probabilities are conditional on surviving another x years. The figure uses the Compustat sample from 1987 until 2014.

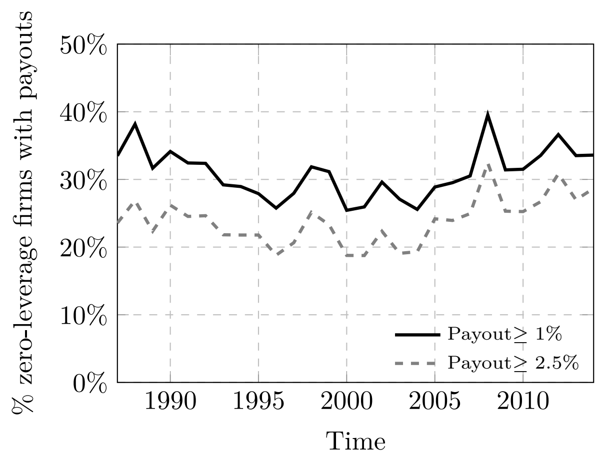


FIGURE 6: **Fraction of zero-leverage firms that make payouts.** The figure plots the fraction of zero-leverage firms (DLTT+DLC=0) that make payouts to shareholders (payouts (PRSTKC+DVC) larger than 1% or 2.5% of assets (AT)) and uses for this the Compustat sample from 1987 until 2014.

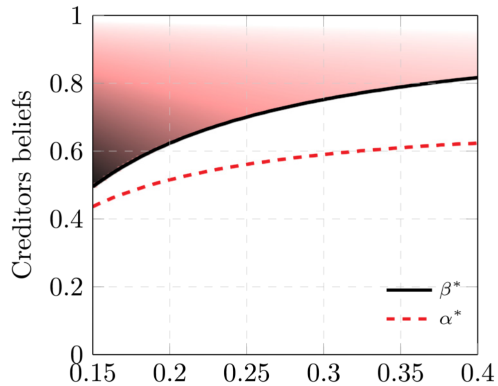
figures show the impact on the issuance thresholds and optimal debt maturity of varying the model parameters. In all cases the issuance region $\mathcal{B}^* = [\beta^*, 1]$. Changing any of the model parameters influences both the timing of debt issuance and the optimal debt maturity. Therefore, changing a model parameter influences the optimal debt maturity at β^* in two ways. First, the timing of debt issuance changes and therefore β^* changes, which influences the optimal debt maturity. Second, the trade-off between issuance costs and adverse selection changes and thus the optimal debt maturity changes. For clarity, I discuss separately the change in the timing of debt issuance, that is changes in (α^*, β^*), and changes in the optimal debt maturity for given beliefs above β^*.

Larger issuance cost q make debt issuance less profitable. Therefore, shareholders of a high-quality firm delay debt issuance and β^* increases. In response, shareholders of a low-quality firm increase α^* such that they are again indifferent between issuance and postponing issuance at α^*. Because separating becomes less profitable for shareholders of a low-quality firm, the difference $\beta^* - \alpha^*$ increases. For given beliefs above β^*, the debt maturity is increasing in the issuance cost because they become more important relative to the adverse selection cost.

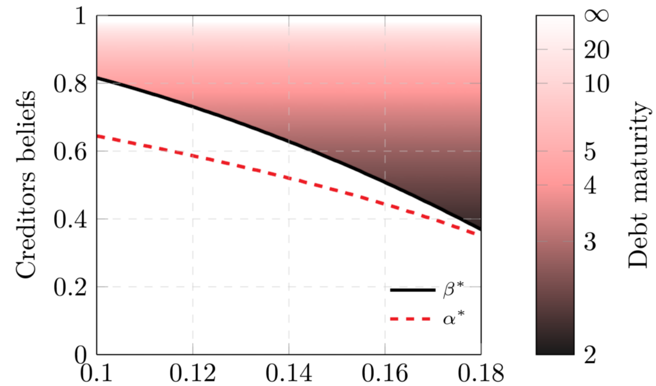
A higher tax benefits to debt π, increase the cost of delaying debt issuance. Therefore, shareholders of a high-quality firm issue debt sooner and the upper issuance threshold β^* decreases. In response, shareholders of a low-quality firm decrease α^* to remain indifferent between issuance and postponing issuance. Because postponing debt issuance also becomes costlier for shareholders of a low-quality firm, the difference $\beta^* - \alpha^*$ decreases. Given beliefs above β^*, the optimal debt maturity decreases in the tax benefits because higher tax benefits increase the benefits from debt issuance which exacerbates the adverse selection problem.

Increasing the interest rate r speeds up debt issuance, although the effects are quantitatively small. Shareholders speed up debt issuance because they become more impatient and value future increases in the debt price less. The speed up of debt issuance occurs mainly through an increase in α^* but also through a small decrease in β^*. For given beliefs above β^*, the optimal debt maturity increases in the interest rate because shareholders value less the option to issue debt at a better price in the future.

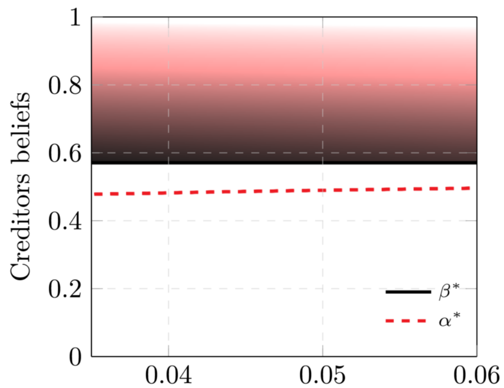
Increasing the failure intensity of a high-quality firm λ_h narrows the wedge between a high and low-quality firm and decreases the informativeness of the firm's survival. Therefore, there are less incentives to delay debt issuance, which decreases β^* and $\beta^* - \alpha^*$. For given beliefs above β^*, the optimal debt maturity is increasing in a high-quality firm's failure intensity because the adverse selection problem becomes less severe and the firm's survival is less informative while the issuance cost remain constant. Fig. 7(e) shows the effects of increasing the failure intensity of a low-quality firm λ_l and yields the opposite results.



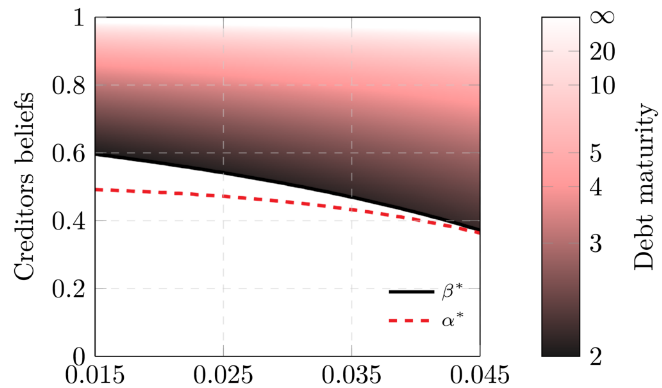
(a) Issuance cost q



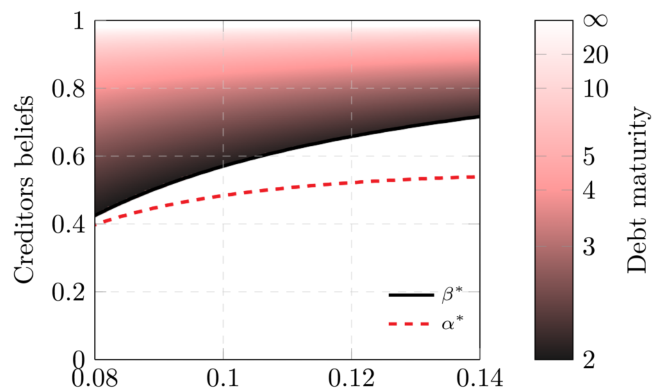
(b) Tax benefits π



(c) Interest rate r



(d) Failure intensity of a h-firm λ_h



(e) Failure intensity of a l-firm λ_l

FIGURE 7: **Comparative statics analysis with respect to the issuance cost q, tax benefits π, interest rate r, and failure intensities λ_h and λ_l.** The figure shows the equilibrium issuance thresholds β^* and α^* and equilibrium debt maturity creditors offer (shaded area). The base case parameters are taken from Table 1, and I vary them one by one.

D. *Efficiency*

In the final part of this section, I study the effects of the firm having the ability to manage its debt maturity on the efficiency of the equilibrium outcome relative to the first-best solution, which is when the firm directly issues long-term debt. Section II shows that in the absence of asymmetric information the first-best solution would be to directly issue long-term debt to minimize issuance costs. Therefore, I define the percentage efficiency loss, relative to first-best, given creditors beliefs p as

$$L(p) = 1 - \frac{pE_h(p) + (1-p)E_l(p)}{\frac{p}{r+\lambda_h} + \frac{1-p}{r+\lambda_l} - q} \in [0, 1].$$

The numerator is the expected value of a (currently) unlevered firm in the presence of asymmetric information, and the denominator is the expected first-best firm value in a world without asymmetric information. This efficiency loss can be interpreted as the costs of asymmetric information.

Fig. 8 plots efficiency loss against beliefs. For the base case solution, the solid line, efficiency loss and beliefs have an inverted-U relationship. When creditors are certain the firm is either high or low quality, beliefs p are close to zero or one, asymmetric information plays almost no role and the efficiency loss is close to zero. For intermediate beliefs, when creditors are uncertain about the quality of the firm, the efficiency loss is largest and therefore the costs of asymmetric information are largest.

Fig. 8 also plots the efficiency loss in case the firm is restricted to issue debt with a fixed maturity, see the grey dashed and dotted lines. When creditors are certain the firm is high or low quality, the efficiency loss is decreasing in the debt maturity because issuance costs play an important role while asymmetric information does not. When asymmetric information is less severe, longer maturity debt is preferred because it lowers the expected amount of times the firm has to pay issuance costs. For intermediate beliefs, when creditors are uncertain about the quality of the firm, the results are different: the efficiency loss first decreases in the debt maturity and then increases. The reason the efficiency loss initially decreases in the debt maturity is because longer maturity debt needs to be rolled over less frequently, which lowers the expected future issuance costs and therefore improves efficiency. At some point,

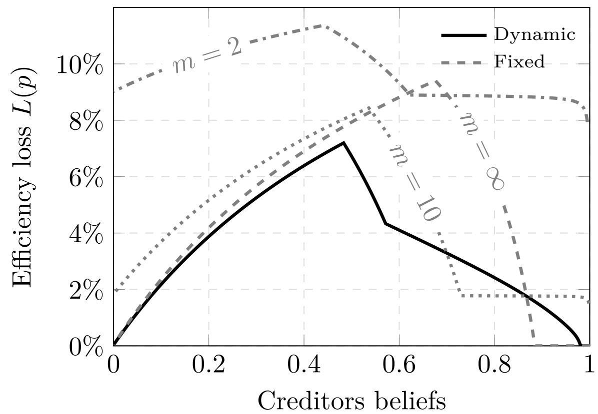


FIGURE 8: **Efficiency loss for firms that have a dynamic and fixed debt maturity choice.** The solid line depicts the efficiency loss in case the firm can dynamically adjust its debt maturity while the grey dashed and dotted lines depict the cases in which the firm can only issue debt with a fixed maturity.

when the firm starts issuing longer maturity debt, asymmetric information starts playing a more important role and this eventually leads to efficiency losses. This again illustrates the main trade-off that determines the optimal debt maturity, namely that of debt issuance costs versus adverse selection costs.

The figure also shows that giving the firm the opportunity to dynamically manage its debt maturity can both improve and harm efficiency. The reason is that both high and low-quality firms set their optimal debt maturity such as to maximize the value of their own claim. When creditors think that the firm is likely to be low quality, giving it the ability to endogenously determine its debt maturity improves efficiency. The reason is that the firm's ability to shorten its debt maturity, and thereby alleviating adverse selection, speeds up debt issuance, which improves efficiency. When creditors are almost certain the firm is of high quality the results are reversed if the firm can only issue debt that has a long enough maturity. The reason is that high-quality firms issue inefficiently short-maturity debt to alleviate the adverse selection problem, while from an efficiency point of view it would be better to issue long-maturity debt to minimize issuance costs. These results illustrate that giving individual firms the ability to overcome adverse selection can be a blessing and a curse.

V. Exogenous News

In this section, I numerically solve the model with exogenous Brownian news as in Daley and Green (2012).[20] The model's predictions with respect to firms' debt maturity choice and zero-leverage firms are robust to this extension. However, the news process influences the dynamics of beliefs, as page 14 shows, and leads to new empirical predictions.

A new result in this setup is that a levered firm can become a zero-leverage firm. This can happen when the debt matures while beliefs are below the upper issuance threshold. Fig. 3(a) gives an example of such a situation, the firm goes from levered to zero leverage at $\tau_1^* + \tau_{\mathcal{M}(\beta)}$. This result is consistent with Fig. 5, which shows empirically that a levered firm has a positive (but small) probability of becoming a zero-leverage firm.

I numerically solve for an equilibrium α, $\mathcal{B}$, and $\mathcal{M}$ using the base case parameters from Table 1. Fig. 9 performs a comparative statics analysis with respect to the news quality. Observe that above the upper issuance threshold the optimal debt maturity is increasing in beliefs. Therefore, the life-cycle theory of debt maturity is robust to adding an exogenous Brownian news process to the model. Furthermore, between the two issuance thresholds the firm postpones debt issuance and has zero leverage.

Changing the news quality ϕ influences both the optimal debt maturity and the issuance thresholds. First, a higher news quality increases a high-quality firm's shareholders incentive to delay debt issuance because the debt price improves at a faster rate. Therefore, they increase β. A higher news quality also causes beliefs to become more volatile because creditors are more responsive to news. This gives shareholders of a low-quality firm a larger incentive to postpone debt issuance, and thus they decrease α. Above the upper issuance threshold, the optimal debt maturity decreases in the news quality because a higher news quality causes creditors to learn faster about the quality of the firm, which increases the option value of issuing debt in the future.

[20] In an earlier version of this article, I show equilibrium existence in a debt issuance model where creditors learn from Brownian news and the debt maturity is fixed. The constructed equilibrium is the same two-threshold equilibrium as described in this article, and I can establish that $\mathcal{B}$ is of the form $[\beta, 1]$. The proof is available upon request.

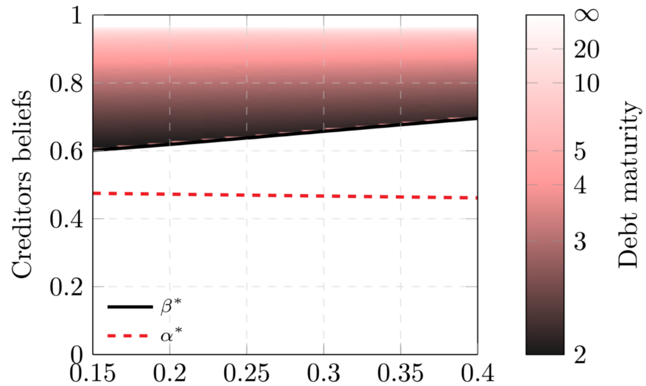


FIGURE 9: **Comparative statics analysis with respect to the news intensity ϕ.** The figure shows the equilibrium issuance thresholds β^* and α^* and equilibrium debt maturity creditors offer (shaded area). The base case parameters are taken from Table 1.

VI. Conclusion

I develop a dynamic capital structure model in which shareholders decide on the timing and maturity of debt issuance. The model leads to a life-cycle theory of debt maturity. Younger firms that face more asymmetric information issue shorter maturity debt to alleviate adverse selection, while mature firms that face less asymmetric information issue longer maturity debt to minimize issuance costs. This life-cycle theory of debt maturity rationalizes the findings in Barclay and Smith (1995), Berger et al. (2005), and Custódio et al. (2013) that firms facing more asymmetric information issue shorter maturity debt and Fig. 1(a)'s finding that debt maturity is positively correlated with firm age. The endogenous timing of debt issuance leads to firms postponing debt issuance to allow the market to learn about their quality in the hope of getting a better debt price in the future. When firms postpone debt issuance they have zero leverage. Furthermore, these zero-leverage firms are expected to issue debt in the future, which is consistent with Korteweg (2010)'s findings.

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Appendix

In this appendix, I solve the base case model without Brownian news. I generalize the model by adding a Poisson news process. This news process has dynamics

$$dI_t = N_t(\kappa_\theta),$$

where κ_θ is the type dependent news intensity. I assume that a news jump is good news, $\kappa_h \geq \kappa_l$. Therefore, if creditors receive a news jump they adjust their beliefs upward. I also assume that survival is more informative than news, $\kappa_h - \kappa_l \leq \lambda_l - \lambda_h$, which induces beliefs to drift upward. When $\kappa_h = \kappa_l$, we are back in the base case without Poisson news.

Appendix A discusses the dynamics of beliefs of $\mathcal{S}(\alpha, \mathcal{B}, \mathcal{M})$. Appendix B contains the equilibrium existence proof (Theorem 1). Finally, Appendix C proofs some of the model's implications (Proposition 2 and Proposition 4).

To simplify notation

$$\begin{aligned}\bar{D}_\theta &= D_\theta(\infty),\\ \bar{K}_\theta &= K_\theta(\infty).\end{aligned}$$

The equity value for an optimally levered firm of quality θ and an unlevered firm of quality θ are

$$\begin{aligned}\bar{D}_\theta - q = D_\theta(\infty) - q &= \mathbb{E}_0^\theta\left[\int_0^{\tau_F} e^{-rt}dt\right] - q,\\ \bar{K}_\theta = K_\theta(\infty) &= \mathbb{E}_0^\theta\left[\int_0^{\tau_F} e^{-rt}(1-\pi)dt\right].\end{aligned}$$

The no deals condition for a low-quality firm boils down to

$$E_l(p) \geq \sup_m \left\{D_l(m) + \mathbb{E}_p^l\left[\mathbb{1}_{\{\tau_m < \tau_F\}}e^{-r\tau_m}E_l(0)\right] - q\right\} = \bar{D}_l - q. \tag{6}$$

A. Dynamics of Beliefs

The first step is to describe creditors' beliefs. From equation (3) and the characteristics of $\mathcal{S}(\alpha, \mathcal{B}, \mathcal{M})$ it follows that creditors' beliefs satisfy the following stochastic differential equation,

$$dP_t = \underbrace{(\lambda_l - \lambda_h)P_t(1-P_t)dt}_{\textit{Survival}} + \underbrace{(\kappa_l - \kappa_h)P_t(1-P_t)dt + f(P_{t^-})dN_t(\kappa_\theta)}_{\textit{Learning from news}} + \underbrace{\alpha\left((1-\alpha)dL_t - dN_t^I\right)}_{\textit{Issuance strategy at or below } \alpha}, \tag{7}$$

where N_t^I is a process that jumps when the firm issues debt at or below α. The first term follows from the firm's survival, the second term from the news process, and the third term from the issuance strategy of a low-quality firm at or below α.

Because beliefs are a martingale with respect to the issuance strategy, I construct the jump intensity of N_t^I to ensure that

$$\mathbb{E}_p\left[(1-\alpha)dL_t - dN_t^I\right] = 0$$

is a martingale. See, also page 21 where I explicitly construct the issuance strategy of a low-quality firm at or below α.

Given creditors' beliefs, the probability of a news jump the next instant is

$$\mathbb{E}_p\left[dN_t(\kappa_\theta)\right] = \left(p\kappa_h + (1-p)\kappa_l\right)dt.$$

Because beliefs are a martingale with respect to the news process I_t, the second term in equation (7) is a martingale,

$$0 = (\kappa_l - \kappa_h)p(1-p) + f(p)\left(p\kappa_h + (1-p)\kappa_l\right).$$

This equality implies that

$$f(p) = \frac{(\kappa_h - \kappa_l)p(1-p)}{p\kappa_h + (1-p)\kappa_l}. \tag{8}$$

Therefore, if a jump occurs for a p_- then

$$p = p_- + f(p_-) = \frac{\kappa_h p_-}{\kappa_h p_- + \kappa_l(1-p_-)}.$$

When the firm has debt outstanding, beliefs evolve according to equation (7) minus the issuance strategy at or below α term.

In the rest of the proof, I work with creditors' beliefs assuming they never issue debt at or below α. This implies that when the firm is unlevered beliefs follow from

$$dP_t = (\lambda_l - \lambda_h)P_t(1-P_t)dt + (\kappa_l - \kappa_h)P_t(1-P_t)dt + f(P_{t^-})dN_t(\kappa_\theta) + \alpha(1-\alpha)dL_t. \tag{9}$$

The term N_t^I, which jumps when shareholders issue debt at or below α, drops out.

These beliefs are non-decreasing over time if,

1. News jumps imply upward jumps in beliefs, $f(p) > 0$. From equation (8) it follows that this is the case when
$$\kappa_h \geq \kappa_l.$$
2. Beliefs have a positive drift. From equation (9) it follows that this is the case when
$$\kappa_h - \kappa_l \leq \lambda_l - \lambda_h.$$

I assume the news and failure intensities satisfy these requirements, and therefore beliefs are non-decreasing over time.

B. Endogenous Debt Maturity

Showing existence of an equilibrium $\mathcal{S}(\alpha, \beta, \mathcal{M})$ (Theorem 1) requires several steps:

1. Given α, a contraction mapping argument shows existence of the equity value for a high-quality firm $E_h(p|\alpha)$. Shareholders issuance strategy maximizes a high-quality firm's equity value. From this optimization problem, the optimal issuance strategy, given by the debt maturity $\mathcal{M}(p|\alpha)$ and issuance region $\mathcal{B}_\alpha$, follows. The construction of a high-quality firm's equity value ensures that issuance optimality and the no deals condition hold.

2. Because P_t is non-decreasing over time, the equity value $E_h(p|\alpha)$ and optimal issuance strategy depend only on the equity value and optimal issuance strategy for $p' \geq p$. This implies that for $p \geq \alpha$ the optimal issuance strategy $\mathcal{M}$ and $\mathcal{B}$ is independent of α. This observation also implies that I can rewrite a high-quality firm's equity value as
$$E_h(p|\alpha) = E_h(p|0) + \mathbb{1}_{\{p<\alpha\}} \left(E_h(\alpha|0) - E_h(p|0) \right).$$

3. Given α and the optimal issuance strategy $(\mathcal{B}, \mathcal{M})$, a contraction mapping argument shows existence of the equity value of a low-quality firm $E_l(p|\alpha)$. Furthermore, because the optimal issuance strategy $(\mathcal{B}, \mathcal{M})$ is independent of α and P_t is non-decreasing over time
$$E_l(p|\alpha) = E_l(p|0) + \mathbb{1}_{\{p<\alpha\}} \left(E_l(\alpha|0) - E_l(p|0) \right).$$

4. Lemma 4 establishes that
$$\alpha^* = \sup \left\{ \alpha | E_l(\alpha|\alpha) \leq \bar{D}_l - q \right\}$$
exists, with this I mean $\sup \left\{ \alpha | E_l(\alpha|\alpha) \leq \bar{D}_l - q \right\}$ is non-empty, and that at α^*
$$E_l(\alpha^*|\alpha^*) = \bar{D}_l - q.$$

5. The construction of α^* ensures that
$$E_l(p|\alpha^*) \geq \bar{D}_l - q.$$
This inequality implies that the no deals condition holds for a low-quality firm, see also equation (6). For $p < \alpha^*$,
$$E_l(p|\alpha^*) = \bar{D}_l - q,$$
and thus shareholders of a low-quality firm are indifferent between issuance and postponing issuance. For $p \notin \mathcal{B} \cap (\alpha^*, 1]$, shareholders of a low-quality firm optimally postpone debt issuance because
$$E_l(p|\alpha^*) > \bar{D}_l - q.$$
For $p \in \mathcal{B} \cap (\alpha^*, 1]$, shareholders optimally issue debt because of creditors threath (Assumption 2) that if shareholders refuse the offer then beliefs become $p_+ = 0$. In that case, the equity value would drop to $\bar{D}_l - q = E_l(0|\alpha^*) \leq E_l(p|\alpha^*)$. Therefore, shareholders of a low-quality firm's issuance strategy is optimal.

6. The zero profit condition and belief consistency hold by construction. Therefore, $\mathcal{S}(\alpha^*, \mathcal{B}^*, \mathcal{M}^*)$ is an equilibrium.

7. Finally, at α^* shareholders of a high-quality firm strictly prefer to abstain from debt issuance and therefore $\alpha^* \notin \mathcal{B}$. This result also implies that the region in which shareholders postpone debt issuance $\mathcal{R} = (\alpha^*, 1) \cap (\mathcal{B}^*)^c$ is non-empty.

Define a high-quality firm's mapping $\mathcal{V}_h(E, \alpha)$ as

$$\begin{aligned}\mathcal{V}_h(E,\alpha) = \sup_{\tau, m \geq \underline{m}_h} \Bigg\{ & \mathbb{E}_p^h \left[\int_0^{\tau \wedge \tau_F} e^{-rt}(1-\pi)dt \right] \\ & + \mathbb{E}_p^h \left[\mathbb{1}_{\{\tau < \tau_F\}} e^{-r\tau} \left\{ \Phi(P_{\tau^-}, m) + \mathbb{1}_{\{\tau + \tau_m < \tau_F\}} e^{-r\tau_m} E(P_{\tau+\tau_m^-}) - q \right\} \right] \Bigg\},\end{aligned}$$

where the dynamics of P_t before debt issuance follow

$$dP_t = (\lambda_l - \lambda_h)P_t(1-P_t)dt + (\kappa_l - \kappa_h)P_t(1-P_t)dt + f(P_{t^-})dN_t(\kappa_h) + \alpha(1-\alpha)dL_t$$

and after debt issuance follow

$$dP_t = (\lambda_l - \lambda_h)P_t(1-P_t)dt + (\kappa_l - \kappa_h)P_t(1-P_t)dt + f(P_{t^-})dN_t(\kappa_h).$$

Proposition 1 shows that issuing debt with a maturity smaller than $\underline{m}_h$ is negative NPV, and I can therefore restrict the firm to choose maturities larger than $\underline{m}_h$.

Lemma 1. *The mapping $\mathcal{V}_h(E, \alpha)$ has a unique fixed point. This fixed point is a continuous function that is bounded between $[\bar{K}_h, \bar{D}_h - q]$.*

Proof. First, any fixed point is a function bounded between $[\bar{K}_h, \bar{D}_h - q]$ since the equity value is bounded from above by a high-quality firm's equity value under perfect information $\bar{D}_h - q$ and from below by the unlevered equity value $\bar{K}_h$.

Second, the mapping $\mathcal{V}_h$ is a contraction in the L_∞-norm,

$$\begin{aligned}& \Big\| \mathcal{V}_h(E_1,\alpha) - \mathcal{V}_h(E_2,\alpha) \Big\| \\ \leq & \sup_{\tau, m \geq \underline{m}_h} \Bigg\| \mathbb{E}_p^h \left[\int_0^{\tau \wedge \tau_F} e^{-rt}(1-\pi)dt \right] \\ & \quad + \mathbb{E}_p^h \left[\mathbb{1}_{\{\tau < \tau_F\}} e^{-r\tau} \left\{ \Phi\left(P_{\tau^-}, m\right) + \mathbb{1}_{\{\tau + \tau_m < \tau_F\}} e^{-r\tau_m} E_1\left(P_{\tau+\tau_m^-}\right) - q \right\} \right] \\ & \quad - \mathbb{E}_p^h \left[\int_0^{\tau \wedge \tau_F} e^{-rt}(1-\pi)dt \right] \\ & \quad - \mathbb{E}_p^h \left[\mathbb{1}_{\{\tau < \tau_F\}} e^{-r\tau} \left\{ \Phi\left(P_{\tau^-}, m\right) + \mathbb{1}_{\{\tau + \tau_m < \tau_F\}} e^{-r\tau_m} E_2\left(P_{\tau+\tau_m^-}\right) - q \right\} \right] \Bigg\| \\ = & \sup_{\tau, m \geq \underline{m}_h} \Bigg\| \mathbb{E}_p^h \left[e^{-r\tau} \mathbb{1}_{\{\tau + \tau_m < \tau_F\}} \left\{ E_1\left(P_{\tau+\tau_m^-}\right) - E_2\left(P_{\tau+\tau_m^-}\right) \right\} \right] \Bigg\| \\ \leq & \frac{\frac{1}{\underline{m}_h}}{r + \lambda_h + \frac{1}{\underline{m}_h}} \|E_1 - E_2\|.\end{aligned}$$

This ensures that the fixed point of $\mathcal{V}_h(E,\alpha)$ exists, is unique, and continuous. $\square$

Define $E_h(p|\alpha)$ as the fixed point of $\mathcal{V}_h(E,\alpha)$ with for $p<\alpha$ $E_h(p|\alpha)=E_h(\alpha|\alpha)$. The optimal issuance strategy is

$$\mathcal{B}_\alpha = \left\{ p \middle| E_h(p|\alpha) = \sup_{m\geq \underline{m}_h} \mathbb{E}^h_p\left[\Phi(p,m) + \mathbb{1}_{\{\tau_m<\tau_F\}} e^{-r\tau_m} E_h\left(P_{\tau_m^-}|\alpha\right) - q\right]\right\},$$

$$\mathcal{M}(p|\alpha) \in \arg\sup_{m\geq \underline{m}_h} \mathbb{E}^h_p\left[\Phi(p,m) + \mathbb{1}_{\{\tau_m<\tau_F\}} e^{-r\tau_m} E_h\left(P_{\tau_m^-}|\alpha\right) - q\right].$$

The construction of $E_h(p|\alpha)$ ensures that the no deals condition and issuance optimality hold for shareholders of a high-quality firm.

Because p is a non-decreasing process, when the firm is alive the equity value and optimal issuance strategy at p depend only on the equity value and optimal issuance strategy for $p'>p$. This observation implies that for $p\geq\alpha$ the optimal issuance strategy is independent of α. Define $\mathcal{B}=\mathcal{B}_0$ and $\mathcal{M}(\cdot)=\mathcal{M}(\cdot|0)$. To simplify notation, I use from now onwards $\mathcal{B}$ to indicate $\mathcal{B}\cap(\alpha,1]$. This observation also implies that the equity value satisfies

$$E_h(p|\alpha) = E_h(p|0) + \mathbb{1}_{\{p<\alpha\}}\left(E_h(\alpha|0) - E_h(p|0)\right).$$

The next step is to show that the equity value of a high-quality firm is increasing in creditors' beliefs,

Lemma 2. *The equity value for shareholders of a high-quality firm is strictly increasing in creditors' beliefs p for $p\geq\alpha$ and constant for $p<\alpha$.*

Proof. The fact that the equity value is constant for $p<\alpha$ follows from the reflection of P_t at α.

Start from a non-decreasing function $E(p)\geq \bar{K}_h$ then $\mathcal{V}_h(E|\alpha)$ is strictly increasing in p because the payoff when shareholders issue debt,

$$\sup_{m\geq m_h}\left\{\Phi(p,m) + \mathbb{E}^h_p\left[\mathbb{1}_{\{\tau_m<\tau_F\}} e^{-r\tau_m} E(P_{\tau_m})\right] - q\right\},$$

is strictly increasing in p. For any sample path of news and failure, the payoff given a higher intial prior strictly dominates that of a lower prior if the higher prior mimics the issuance time of the lower prior. In expectation, the optimal issuance strategy given the higher prior dominates this suboptimal issuance strategy. Therefore, $\mathcal{V}_h(E|\alpha)$ must be increasing in p. Induction finishes the proof. $\square$

Define a low-quality firm's mapping $\mathcal{V}_l(E,\alpha)$ as

$$\begin{aligned}\mathcal{V}_l(E,\alpha) =& \mathbb{E}_p^l\left[\int_0^{\tau_{\mathcal{B}}\wedge\tau_F} e^{-rt}(1-\pi)dt\right] \\ &+ \mathbb{E}_p^l\left[\mathbb{1}_{\{\tau_{\mathcal{B}}<\tau_F\}}e^{-r\tau}\left\{\Phi\left(P_{\tau_{\mathcal{B}}^-},\mathcal{M}\left(P_{\tau_{\mathcal{B}}^-}\right)\right)+\mathbb{1}_{\left\{\tau_{\mathcal{B}}+\tau_{\mathcal{M}\left(P_{\tau_{\mathcal{B}}^-}\right)}<\tau_F\right\}}e^{-r\tau_m}E\left(P_{\tau_{\mathcal{B}}+\tau_{\mathcal{M}}^-\left(P_{\tau_{\mathcal{B}}^-}\right)}\right)-q\right\}\right],\end{aligned}$$

where the dynamics of P_t before debt issuance follow

$$dP_t = (\lambda_l-\lambda_h)P_t(1-P_t)dt + (\kappa_l-\kappa_h)P_t(1-P_t)dt + f(P_{t^-})dN_t(\kappa_l) + \alpha(1-\alpha)dL_t$$

and after debt issuance follow

$$dP_t = (\lambda_l-\lambda_h)P_t(1-P_t)dt + (\kappa_l-\kappa_h)P_t(1-P_t)dt + f(P_{t^-})dN_t(\kappa_l),$$

and the stopping time $\tau_{\mathcal{B}} = \inf\{t>0|P_t\in\mathcal{B}\}$.

Lemma 3. *The mapping $\mathcal{V}_l(E,\alpha)$ has a unique fixed point. This fixed point is a real-valued function that is bounded between $[\bar{K}_l,\bar{D}_h-q]$.*

Proof. The same arguments as in Lemma 1 show that $\mathcal{V}_l$ is a contraction mapping. The lower-bound is replaced by $\bar{K}_l$. Shareholders of a high-quality firm only issue debt if it is positive NPV and thus

$$K_l(m) < K_h(m) < \Phi(p,m) - q,$$

which ensures that the firm is worth more than the unlevered firm value $\bar{K}_l$. Furthermore, the fixed point is not necessariliy continuous in p because $\mathcal{B}$ and $\mathcal{M}$ are given and do not maximize shareholders of a low-quality firm's equity value, as is the case for the mapping $\mathcal{V}_h$. □

Define the fixed point of the mapping $\mathcal{V}_l(E,\alpha)$ as $E_l(p|\alpha)$ with for $p<\alpha$ $E_l(p|\alpha) = E_l(\alpha|\alpha)$. Because the issuance strategy of shareholders of a high-quality firm is independent of α and beliefs P_t are non-decreasing

$$E_l(p|\alpha) = E_l(p|0) + \mathbb{1}_{\{p<\alpha\}}\left(E_l(\alpha|0) - E_l(p|0)\right).$$

Lemma 4. *There exists a constant α^* such that*

$$\alpha^* = \sup\left\{\alpha|E_l(\alpha|\alpha) \le \bar{D}_l - q\right\}$$

with

$$E_l(\alpha^*|\alpha^*) = \bar{D}_l - q.$$

Proof. The proof has several steps:

1. The fact that the issuance strategy is independent of α and beliefs P_t are non-decreasing implies that the result is equivalent to showing that an α^* such that

$$\alpha^* = \sup \left\{ \alpha | E_l(\alpha|0) \leq \bar{D}_l - q \right\}$$

with

$$E_l(\alpha^*|0) = \bar{D}_l - q$$

exists.

2. Assumption 1 ensures that issuing debt at a low-quality firm's valuation is suboptimal for shareholders of a high-quality firm,

$$K_h(m) > D_l(m) - q \quad \forall m > \underline{m}_h.$$

This assumption then implies that there exists a lower bound,

$$\underline{p} = \sup \left\{ p | K_h(m) \geq \Phi(p, m) - q \quad \forall m > \underline{m}_h \right\} > 0,$$

such that shareholders of a high-quality firm would never issue debt for $p < \underline{p}$. Therefore, if p goes to zero shareholders of a low-quality firm never issue debt and

$$\lim_{p \to 0} E_l(p|0) = \bar{K}_l.$$

3. The fact that the no deals condition holds by construction for $E_h(p|\alpha)$, $\bar{D}_h - q \geq E_h(p|\alpha)$, and Lemma 2 imply that

$$\lim_{p \to 1} E_h(p|\alpha) = \bar{D}_h - q.$$

4. The next step is to show that there exists a p such that $E_l(p|0) > \bar{D}_l - q$. Define $\hat{M}(m)$, for m sufficiently large, as

$$\hat{M}(m) = \frac{\frac{q}{\bar{D}_h - D_h(m)} - 1}{r + \lambda_h} > 0.$$

The function $\hat{M}(m)$ is increasing in m. Assumption 1 then ensures there exists an $\bar{m} < \infty$ such that

$$K_h(\hat{M}(\bar{m})) > \bar{D}_l - q.$$

Because a high-quality firm's equity value is increasing and its limit is $\bar{D}_h - q$ there exists a $\bar{p}$ such that for $p > \bar{p}$

$$E_h(p|0) > D_h(\bar{m}) - q.$$

If for $p > \bar{p}$ the firm would never issue any debt then

$$E_h(p|0) = \bar{K}_h,$$

because P_t is non-decreasing, which violates the previously found limit and therefore the firm must issue debt at some point. Thus, there exists a $p^* > \bar{p}$ such that shareholders issue debt when P_t reaches p^* and this debt has a maturity m^*. At p^* the equity value of a high-quality firm satisfies

$$E_h(p^*|0) = \underbrace{\Phi(p^*, m^*) - q}_{\textit{Proceeds from debt issuance}} + \underbrace{\mathbb{E}_0^h\left[e^{-r\tau_{m^*}}\mathbb{1}_{\{\tau_{m^*}<\tau_F\}}E_h(P_{\tau_{m^*}}|0)|P_0 = p^*\right]}_{\textit{Levered equity value}},$$

The levered equity value satisfies

$$\begin{aligned}\mathbb{E}_0^h\left[e^{-r\tau_{m^*}}\mathbb{1}_{\{\tau_{m^*}<\tau_F\}}E_h(P_{\tau_{m^*}}|0)|P_0 = p^*\right] &< \mathbb{E}_0^h\left[e^{-r\tau_{m^*}}\mathbb{1}_{\{\tau_{m^*}<\tau_F\}}\left(\bar{D}_h - q\right)|P_0 = p^*\right]\\ &= \frac{\frac{1}{m^*}}{r + \lambda_h + \frac{1}{m^*}}\left(\bar{D}_h - q\right).\end{aligned}$$

The inequality follows from the fact that the equity value under perfect information $\bar{D}_h - q$ dominates the equity value $E_h(p|0)$. The equality follows from integrating out the exponential times τ_F and τ_{m^*}. Therefore,

$$D_h(m^*) - q + \frac{\frac{1}{m^*}}{r + \lambda_h + \frac{1}{m^*}}(\bar{D}_h - q) > E_h(p^*|0) > D_h(\bar{m}) - q.$$

I rewrite the left-hand side as

$$\begin{aligned}&D_h(m^*) - q + \frac{\frac{1}{m^*}}{r + \lambda_h + \frac{1}{m^*}}(\bar{D}_h - q)\\ =&\frac{1}{r + \lambda_h + \frac{1}{m^*}} - q + \frac{\frac{1}{m^*}}{r + \lambda_h + \frac{1}{m^*}}\frac{1}{r + \lambda_h} - \frac{\frac{1}{m^*}}{r + \lambda_h + \frac{1}{m^*}}q\\ =&\frac{1}{r + \lambda_h} - q - \frac{\frac{1}{m^*}}{r + \lambda_h + \frac{1}{m^*}}q\\ =&\left(\bar{D}_h - q\right) - \frac{\frac{1}{m^*}}{r + \lambda_h + \frac{1}{m^*}}q\\ >&D_h(\bar{m}) - q\end{aligned}$$

or equivalently

$$\begin{aligned}\bar{D}_h - D_h(\bar{m}) &> \frac{\frac{1}{m^*}}{r + \lambda_h + \frac{1}{m^*}}q,\\ m^* &> \frac{\frac{q}{\bar{D}_h - D_h(\bar{m})} - 1}{r + \lambda_h} = \hat{M}(\bar{m}).\end{aligned}$$

This result combined with the choice of $\bar{m}$ ensures that

$$E_l(p^*|0) > \Phi(p^*, m^*) - q > K_h(m^*) > K_h\left(\hat{M}(\bar{m})\right) > \bar{D}_l - q$$

because $E_l(p^*|0)$ exceeds the debt proceeds $\Phi(p^*, m^*) - q$, which exceed $K_h(m^*)$ because debt issuance is optimal for shareholders of a high-quality firm.

5. The previous two steps ensure that

$$\alpha^* = \sup\left\{\alpha | E_l(\alpha|0) \leq \bar{D}_l - q\right\}$$

exists.

6. The equity value $E_l(p|0)$ is not necessarily continuous in p. If at α^* the equity value jumps then

$$\lim_{p\uparrow\alpha^*} E_l(\alpha|0) \leq \bar{D}_l - q < E_l(\alpha^*|0).$$

For this jump to occur issuance must take place at $\alpha^*_- = \lim_{p\uparrow\alpha^*} p$ else the equity value would be continuous because in a no issuance region the equity value is continuous. The debt issued at α^*_- has expected maturity $M_{\alpha^*_-}$. Debt issuance at α^* then implies that

$$\bar{D}_l - q \geq \Phi(\alpha^*_-, M_{\alpha^*_-}) - q + \mathbb{E}^l_0\left[e^{-r\tau_{M_{\alpha^*_-}}}\mathbb{1}_{\{\tau_{M_{\alpha^*_-}}<\tau_F\}} E_l(P_{\tau_{M_{\alpha^*_-}}}|0)\Big|P_0 = \alpha^*_-\right],$$

$$\bar{D}_l - q > \Phi(\alpha^*_-, M_{\alpha^*_-}) - q + \mathbb{E}^l_0\left[e^{-r\tau_{M_{\alpha^*_-}}}\mathbb{1}_{\{\tau_{M_{\alpha^*_-}}<\tau_F\}}\left(\bar{D}_l - q\right)\Big|P_0 = \alpha^*_-\right],$$

$$\frac{(r+\lambda_l)\left(\bar{D}_l - q\right)}{r + \lambda_l + \frac{1}{M_{\alpha^*_-}}} > \Phi(\alpha^*_-, M_{\alpha^*_-}) - q,$$

$$K_h(M_{\alpha^*_-}) > \frac{r+\lambda_l}{r+\lambda_l+\frac{1}{M_{\alpha^*_-}}}\left(\bar{D}_l - q\right) > \Phi(\alpha^*_-, M_{\alpha^*_-}) - q.$$

The first equation follows from the definition of the levered equity value. The first step follows from the fact that P_t is non-decreasing and that for $p > \alpha^*$ by construction

$$E_l(p|\alpha^*) > \bar{D}_l - q.$$

The second step follows from the fact that both τ_F and $\tau_{M_{\alpha^*_-}}$ are exponentially distributed times. The third step follows from Assumption 1. This result contradicts optimality of issuance at α^* for shareholders a high-quality firm, because these shareholders would be better off postponing debt issuance for $\tau_{M_{\alpha^*_-}}$ time. Therefore, the equity value cannot jump at α^* and

$$E_l(\alpha^*|0) = \bar{D}_l - q.$$

□

Theorem 2. *If a news jump is good news and survival of the firm is more informative than* *the news process,*

$$\kappa_h \geq \kappa_l,$$
$$\kappa_h - \kappa_l \leq \lambda_l - \lambda_h,$$

then there exists a constant α^*, a set $\mathcal{B}^*$, and a function $\mathcal{M}^*$ such that $\mathcal{S}(\alpha^*, \mathcal{B}^*, \mathcal{M}^*)$ is an equilibrium. Furthermore, shareholders postpone debt issuance in some non-empty region above α^*.

Proof. The proof has several steps:

1. Lemma 1 constructs the equity value of a high-quality firm given α, $E_h(p|\alpha)$. By construction, shareholders of a high-quality firm issuance strategy is optimal and their equity value satisfies the no deals condition.

2. The optimal issuance strategy $\mathcal{B}^*$ and $\mathcal{M}^*$ follows from the construction of the equity value of a high-quality firm.

3. Lemma 3 constructs the equity value of a low-quality firm given α, $E_l(p|\alpha)$.

4. Lemma 4 shows existence of an $\alpha^* = \sup\{\alpha | E_l(\alpha|\alpha) \leq \bar{D}_l - q\}$ with

$$E_l(\alpha^*|\alpha^*) = \bar{D}_l - q.$$

5. The construction of α^* ensures that

$$E_l(p|\alpha^*) \geq \bar{D}_l - q.$$

   This inequality implies that the no deals condition holds for a low-quality firm.

6. For $p < \alpha^*$,

$$E_l(p|\alpha^*) = \bar{D}_l - q,$$

   and thus shareholders of a low-quality firm are indifferent between issuance and postponing issuance. For $p \in (\alpha^*, 1] \cap (\mathcal{B}^*)^c$, shareholders of a low-quality firm optimally postpone debt issuance because

$$E_l(p|\alpha^*) \geq \bar{D}_l - q.$$

   For $p \in \mathcal{B}^*$, shareholders optimally issue debt because of creditors threath (Assumption 2) that if shareholders refuse the offer then beliefs become $p_+ = 0$ and $E_l(0|\alpha^*) = \bar{D}_l - q \leq E_l(p|\alpha^*)$. Therefore, shareholders of a low-quality firm's issuance strategy is optimal.

7. The zero profit condition and belief consistency hold by construction, and therefore I have constructed an $\mathcal{S}(\alpha^*, \mathcal{B}^*, \mathcal{M}^*)$ equilibrium.

8. Step 7 of Lemma 4 leads to a contradiction if issuance is optimal at α^* and therefore $\alpha^* \notin \mathcal{B}^*$. Furthermore, this strict inequality holds for any $m > \underline{m}_h$, and therefore in

some right neighborhood of α^* debt issuance is also suboptimal. This implies that $\mathcal{R}$ is non-empty.

$\square$

Theorem 1. *There exists a constant α^*, a set $\mathcal{B}^*$, and a function $\mathcal{M}^*$ such that $\mathcal{S}(\alpha^*, \mathcal{B}^*, \mathcal{M}^*)$ is an equilibrium. Furthermore, shareholders postpone debt issuance in some non-empty region above α^*.*

Proof. Follows directly from Theorem 2. $\square$

C. Model Implications

Proposition 2. *For $p \geq \alpha^*$, there exists a strictly increasing lower bound on the optimal debt maturity $\mathbb{M}(p)$,*

$$\mathcal{M}(p) \geq \mathbb{M}(p).$$

Furthermore, when creditors become more certain that the firm is high-quality

$$\lim_{p \to 1} \mathbb{M}(p) = \infty.$$

Proof. The proof has several steps:

1. Lemma 2 shows that the equity value is strictly increasing in p for $p \geq \alpha^*$ and the proof of Lemma 4 shows that

$$\lim_{p \to 1} E_h(p|\alpha^*) = \bar{D}_h - q.$$

2. This previous step implies that for every p there exists an

$$\mathbb{M}(p) = \left\{ m \in \mathbb{R}_+ \middle| E_h(p|\alpha^*) = \bar{D}_h - q - \frac{\frac{1}{m}}{r + \lambda_h + \frac{1}{m}} q \right\},$$

Furthermore, because the right-hand side is strictly increasing in m and the equity value is strictly increasing in p the function $\mathbb{M}(p)$ is strictly increasing in p.

3. If the firm issues debt with a maturity m' shorter than $\mathbb{M}(p)$, the equity value is

bounded from below by

$$
\begin{aligned}
E_h(p|\alpha^*) &= \bar{D}_h - q - \frac{\frac{1}{\mathbb{M}(p)}}{r+\lambda_h+\frac{1}{\mathbb{M}(p)}} q \\
&> \bar{D}_h - q - \frac{\frac{1}{m'}}{r+\lambda_h+\frac{1}{m'}} q \\
&= \frac{1}{r+\lambda_h+\frac{1}{m'}} - q + \frac{\frac{1}{m'}}{r+\lambda_h+\frac{1}{m'}} \frac{1}{r+\lambda_h} - \frac{\frac{1}{m'}}{r+\lambda_h+\frac{1}{m'}} q \\
&= \frac{1}{r+\lambda_h+\frac{1}{m'}} - q + \frac{\frac{1}{m'}}{r+\lambda_h+\frac{1}{m'}} \left(\bar{D}_h - q\right) \\
&\geq \frac{1}{r+\lambda_h+\frac{1}{m'}} - q + \frac{\frac{1}{m'}}{r+\lambda_h+\frac{1}{m'}} \mathbb{E}_p^h\left[e^{-(r+\lambda_h)\tau_{m'}} E_h(P_{\tau_{m'}}|\alpha^*)\right].
\end{aligned}
$$

The first inequality follows from the fact that $m' < \mathbb{M}(p)$. The second inequality follows from the fact that $\bar{D}_h - q$ dominates the equity value $E_h(p|\alpha^*)$. This result leads to a contradiction.

4. The limiting result follows from the no deals condition that $E_h(p|\alpha^*)$ satisfies.

□

Proposition 4. *For a zero-leverage firm ($p \in \mathcal{R}$), there is a positive probability that the firm issues debt in the future in that*

$$\mathbb{P}_p^\theta(\tau_\theta < \tau_F) > 0.$$

As a result, firm value, equity value for a zero-leverage firm, is larger than unlevered firm value in that

$$\mathbb{E}_p\left[E_\theta(p)\right] > \mathbb{E}_p\left[\int_0^{\tau_F} e^{-rt}(1-\pi)dt\right].$$

Proof. Given P_t, which is increasing over time, there is a $\bar{t}_p$ such that

$$\bar{t}_p = \inf\left\{t > 0 | P_t \geq \inf\left\{\mathcal{B} \cap [p,1)\right\}\right\}.$$

If $\bar{t}_p$ does not exist then $\mathcal{B} \cap [p,1) = \emptyset$ and therefore $E_h(p) = \bar{K}_h$ because the firm would never issue any debt. This result violates the no deals condition and therefore $\mathcal{B} \cap [p,1)$ is non-empty and $\bar{t}_p$ is finite. The probability of debt issuance is therefore at least

$$\mathbb{P}_p^\theta(\tau^* < \tau_F) > \mathbb{P}_p^\theta\left(\mathbb{1}_{\left\{P_{\bar{t}_p}=\inf\{\mathcal{B}\cap[p,1)\}\right\}} * \mathbb{1}_{\{\bar{t}_p<\tau_F\}}\right) > 0.$$

The right-hand side probability is the probability that the firm survives until $\bar{t}_p$ and receives no news jumps until $\bar{t}_p$ in which case it directly issues debt. This probability is for any finite $\bar{t}_p$ strictly positive. There is a positive probability that the firm issues debt, and the equity value reflects this (possible) future change in debt policy. □